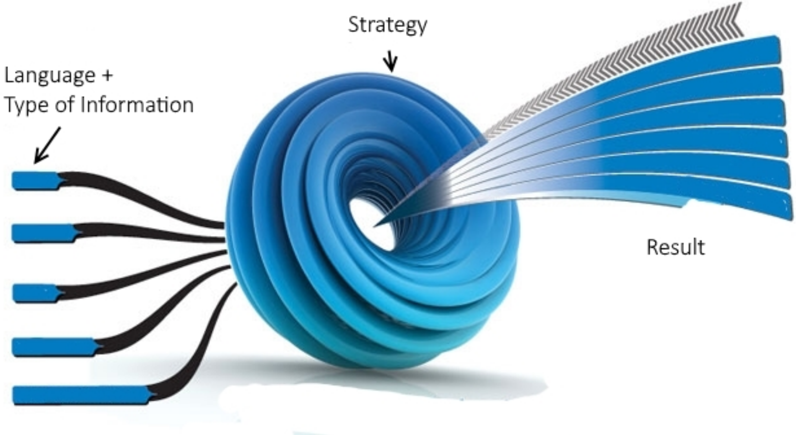
Belief Merging and Distributive Justice

Henrique Viana

Universidade Federal do Ceará - UFC

November 19, 2015

Definition - Information Fusion



Information Fusion

Type of Information

- Belief
- Knowledge
- Goals or Preferences
- Rules, Specifications or Laws

Language

- Propositional Logic
- Horn Logic
- Modal Logic
- First Order Logic
- Multi-Valued Logic

Strategy

- Voting
- Merging
- Coalition
- Auction
- Negotiation
- Cooperation
- Conciliation
- Argumentation

Propositional Belief Merging

Merging Operator with Integrity Constraint

$\Delta_\mu : E \to \mathcal{P}(\Omega)$, where

- E : profile
- Ω : set of interpretations
- μ : integrity constraint

Profile

A profile $E = \{K_1, \ldots, K_n\}$ represents sets of belief bases

Integrity Constraint

A propositional formula μ which the result of merging has to obey

Propositional Belief Merging

Model-based Merging Operator with Integrity Constraint

$\Delta_{\mu}^{d,f} : E \rightarrow \mathcal{P}(\Omega)$, where

- E : profile
- Ω : set of interpretations
- d : distance measure
- f : aggregation function
- μ : integrity constraint

Example

- propositional variables: s, d and o
- set of interpretations: $\Omega = \{\omega_1, \ldots, \omega_8\}$, where: $\omega_1 = \neg s \neg d \neg o$, $\omega_2 = \neg s \neg do$, $\omega_3 = \neg sd \neg o$, $\omega_4 = \neg sdo$, $\omega_5 = s \neg d \neg o$, $\omega_6 = s \neg do$, $\omega_7 = sd \neg o$ and $\omega_8 = sdo$

Distance

A distance measure between interpretations is a total function d from $\Omega \times \Omega$ to $\mathbb{N}$ such that for every $\omega_1, \omega_2 \in \Omega$,

- $d(\omega_1, \omega_2) = d(\omega_2, \omega_1)$, and
- $d(\omega_1, \omega_2) = 0$ if and only if $\omega_1 = \omega_2$

Example

- Hamming Distance: d_H

The Hamming distance between $\omega_1 = \neg s \neg d \neg o$ and $\omega_6 = s \neg d o$ is $d_H(\omega_1, \omega_6) = 2$

Aggregation Function

Examples of aggregation functions: sum, max, generalized max

Example - Languages SQL, O_2 and $Datalog$

$K_1 = (s \vee o) \wedge \neg d$, $K_2 = (\neg s \wedge d \wedge \neg o) \vee (\neg s \wedge \neg d \wedge o)$ and $K_3 = (s \wedge d \wedge o)$

- Merging Operator: $\Delta_{\mu}^{d_H,sum}(E)$, where $\mu = \top$

Ω	$d_H(\omega, K_1)$	$d_H(\omega, K_2)$	$d_H(\omega, K_3)$	$\Delta_{\mu}^{d_H,sum}$
$\omega_1 = \neg s \neg d \neg o$	1	1	3	5
$\omega_2 = \neg s \neg d o$	0	0	2	2
$\omega_3 = \neg s d \neg o$	2	0	2	4
$\omega_4 = \neg s d o$	1	1	1	3
$\omega_5 = s \neg d \neg o$	0	2	2	4
$\omega_6 = s \neg d o$	0	1	1	2
$\omega_7 = s d \neg o$	1	1	1	3
$\omega_8 = s d o$	1	2	0	3

$$\Delta_{\mu}^{d_H,sum}(E) = \omega_2 \vee \omega_6 = (\neg s \wedge \neg d \wedge o) \vee (s \wedge \neg d \wedge o)$$

Logical Properties

- **(IC0)** $\Delta_\mu(E) \models \mu$
- **(IC1)** If μ is consistent, then $\Delta_\mu(E)$ is consistent
- **(IC2)** If $\bigwedge E$ is consistent with μ, then $\Delta_\mu(E) \equiv \bigwedge E \wedge \mu$
- **(IC3)** If $E_1 \equiv E_2$ and $\mu_1 \equiv \mu_2$, then $\Delta_{\mu_1}(E_1) \equiv \Delta_{\mu_2}(E_2)$
- **(IC4)** If $K_1 \models \mu$ and $K_2 \models \mu$, then $\Delta_\mu(\{K_1, K_2\}) \wedge K_1$ is consistent if and only if $\Delta_\mu(\{K_1, K_2\}) \wedge K_2$ is consistent
- **(IC5)** $\Delta_\mu(E_1) \wedge \Delta_\mu(E_2) \models \Delta_\mu(E_1 \sqcup E_2)$
- **(IC6)** If $\Delta_\mu(E_1) \wedge \Delta_\mu(E_2)$ is consistent, then $\Delta_\mu(E_1 \sqcup E_2) \models \Delta_\mu(E_1) \wedge \Delta_\mu(E_2)$
- **(IC7)** $\Delta_{\mu_1}(E) \wedge \mu_2 \models \Delta_{\mu_1 \wedge \mu_2}(E)$
- **(IC8)** If $\Delta_{\mu_1}(E) \wedge \mu_2$ is consistent, then $\Delta_{\mu_1 \wedge \mu_2}(E) \models \Delta_{\mu_1}(E)$

Logical Properties

- **(Maj)** $\exists n \Delta_\mu(E_1 \sqcup \underbrace{E_2 \sqcup \cdots \sqcup E_2}_{n}) \models \Delta_\mu(E_2)$.

- **(Arb)** If

$$\begin{aligned} \Delta_{\mu_1}(\{K_1\}) &\equiv \Delta_{\mu_2}(\{K_2\}) \\ \Delta_{\mu_1 \leftrightarrow \neg\mu_2}(\{K_1, K_2\}) &\equiv (\mu_1 \leftrightarrow \neg\mu_2) \\ \mu_1 &\not\models \mu_2 \\ \mu_2 &\not\models \mu_1 \end{aligned}$$

  Then

$$\Delta_{\mu_1 \vee \mu_2}(\{K_1, K_2\}) \equiv \Delta_{\mu_1}(\{K_1\}).$$

Some Issues about Belief Merging

- Improve the quality of the merging
  - **Changes in the notions of distance and aggregation function**
- Capture the variations of the syntax
  - **Definitions sensible to the syntax**
- Grant the logical properties
  - **Good and intuitive functions**

Distributive Justice

Distributive justice concerns the nature of a socially just allocation of goods in a society.

Types of Distributive Norms

1. **Equity:** an individual who has invested a large amount of input should receive more from the group
2. **Equality:** Regardless of their inputs, all group members should be given an equal share of the rewards/costs
3. **Power:** Those with more authority over the group should receive more than those in lower level positions
4. **Need:** Those in greatest needs should be provided with resources needed to meet those needs
5. **Responsibility:** Group members who have the most should share their resources with those who have less

Principles of Distributive Justice

- Utilitarianism
- Egalitarianism
- Sufficientarianism
- Libertarianism
- Welfare-Based Principles
- Desert-Based Principles

Principles of Distributive Justice

- Utilitarianism
  - The best moral action is the one that maximizes utility
  - Ex: sum operator
- Egalitarianism
  - Favors equality for all people
  - Ex: min, leximin operators
  - Fuzzy Operators
- Sufficientarianism
- Libertarianism
- Welfare-Based Principles
- Desert-Based Principles

Work in Progress

1. Parameterized T-norms as egalitarian merging operators
2. Sufficientarian merging operators
3. Variations of Utilitarianism
4. Other important Principle of Distributive Justice
5. Relationships among Principles of Distributive Justice
6. Model-based Merging vs Other Merging