Epistemic Logics for Authentication and Secrecy in Protocols

V Workshop Científico LogIA - PenCogLin

UFC

Mario Benevides

Federal University of Rio de Janeiro - Brazil

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Join work with Anna Oliveira, Luiz Fernandez and Ivan Varzinczak

Overview

- Motivation

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- Algebraic and Logic Approaches for Autheticity and Secrecy

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- Epistemic Logic for Autheticity and Secrecy

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- Epistemic Logic for Autheticity and Secrecy
- Logics for Authorization

Outline

- Modelo de Dolev Yao: On the Security of Public Key Protocols - 1983

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  The content of a message encrypted for agent A can only be read by A

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- Secrecy:

  No agents can read the content of any message which was not encrypted for him

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- **Validade**: **EXPTIME**-Completo,
  - Lógica Dinâmica Proposicional PDL
  - Lógica Epistêmica Multi-agente (c/ Conhecimento Comum)
  - CTL - Computation Tree Logic (Tempotal)

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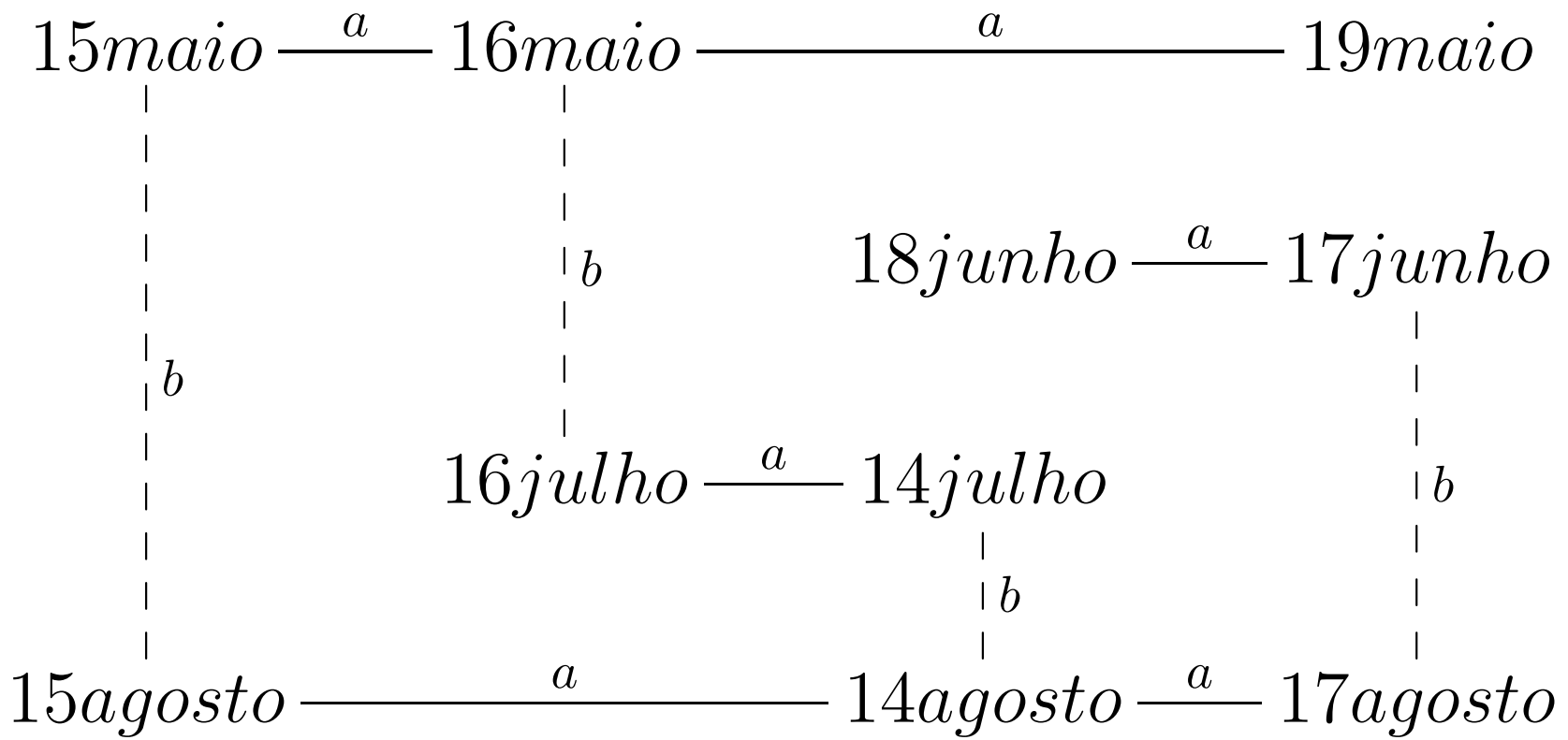

Quebra Cab. Lógicos Beberrões

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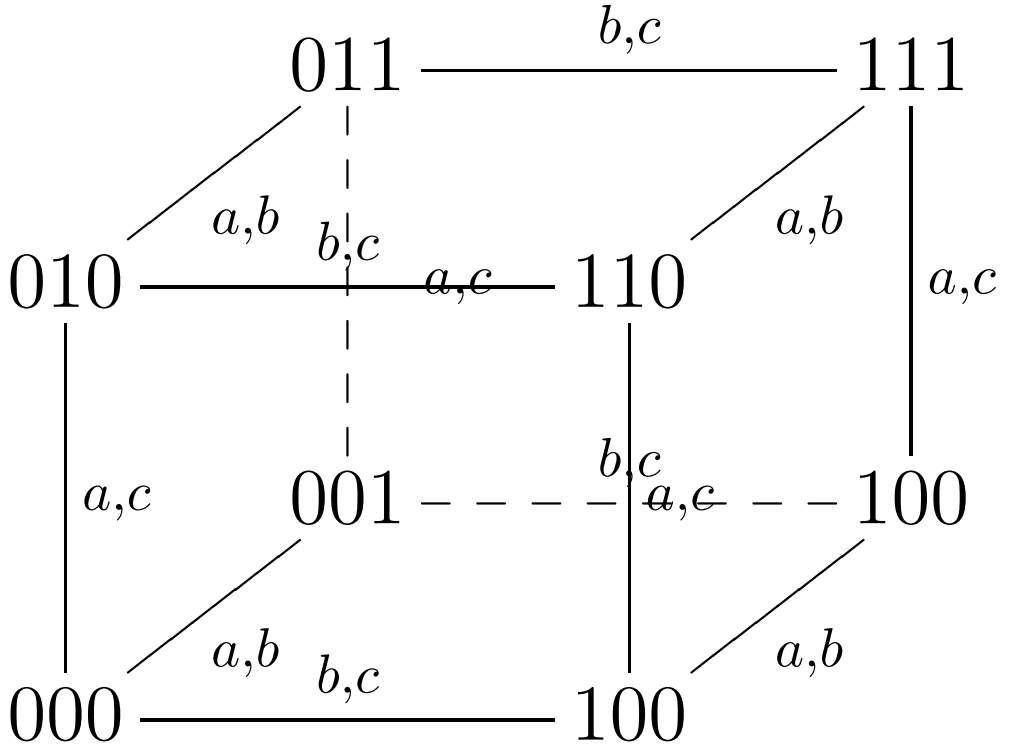
- Como Carla soube???

Quebra Cab. Lógicos Beberrões

- Estado: notação posicional: **a b c**
- Ex.: **011 - a**na quer beber, **b**eto não e **c**arla não;



Procolo

- 3 Agentes: A, B e Z (malicioso)
- **Modelo de Chave Públicas** (Public Key Protocol)

- função de encriptação: E_X (publica), $X = \{A, B, Z\}$
- função de decriptação: D_X (só X sabe), $X = \{A, B, Z\}$
- $D_X E_X(M) = M$

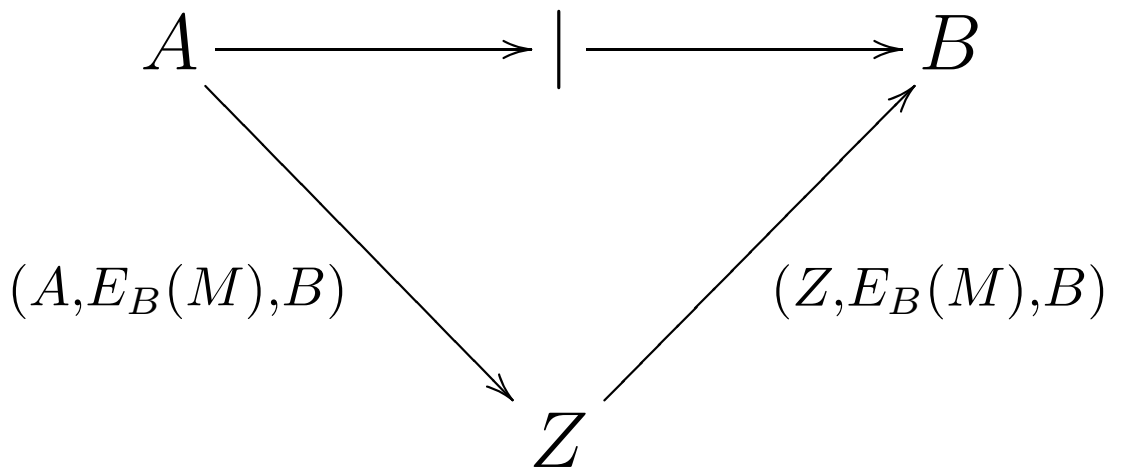
Protocolol

A manda msg M para B

$$A \longrightarrow (A, E_B(M), B) \longrightarrow B$$

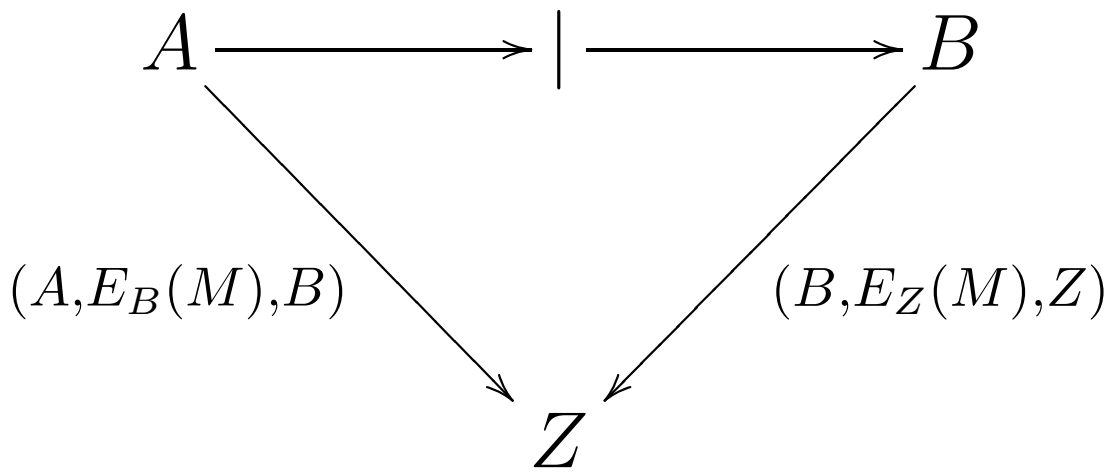
Z intercepta a msg de A para B

Z sends envia $(Z, E_B(M), B)$ para B



Protocolo

B envia msg $(B, E_Z(M), Z)$ para Z



Z decodifica $E_Z(M)$ e obtem M

Como Z raciocinou p/ quebrar o Protocolo?

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- **NOT** Encription Level.

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- encryption function E_X (public)
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- $D_X E_X(M) = M$
- for any user Y knowing $E_X(M)$ does not reveal anything about M

Example 1: Dolev & Yao Model

A sends msg M to B

$$A \longrightarrow (A, E_B(M), B) \longrightarrow B$$

Intruder Z intercepts the message sent from A to B

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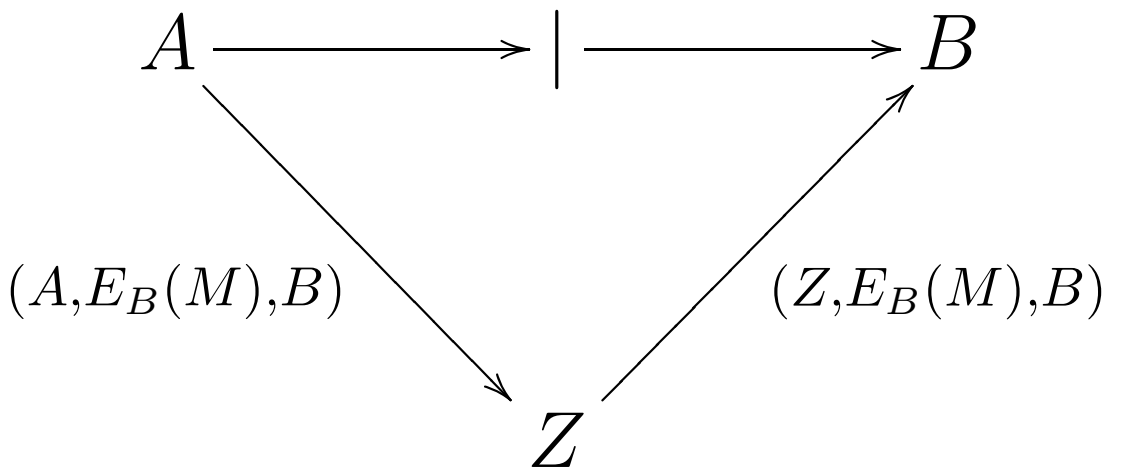

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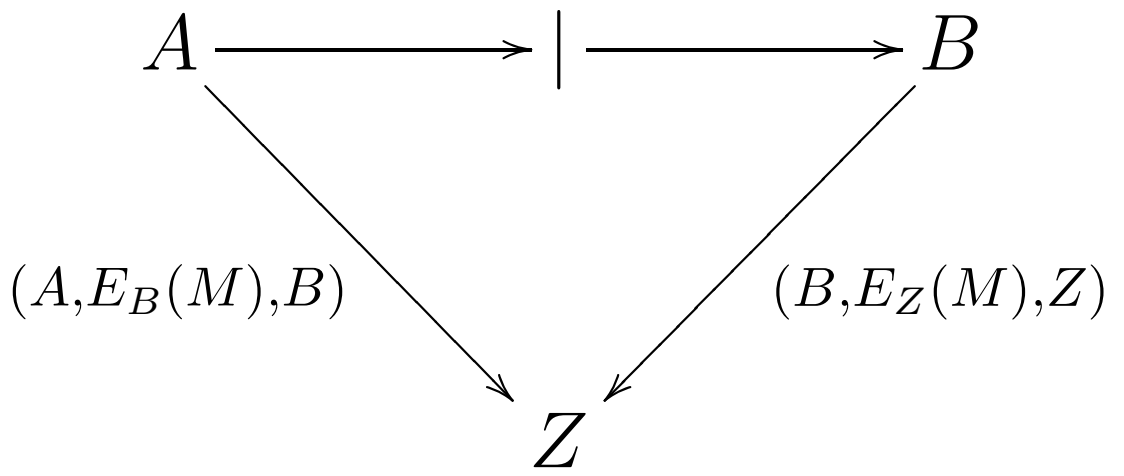

Intruder Z decodes $E_Z(M)$ and obtains M

Example 2: Dolev & Yao Model

A sends msg MA to B and B replies to the user that is encrypted with the message M and not to the sender

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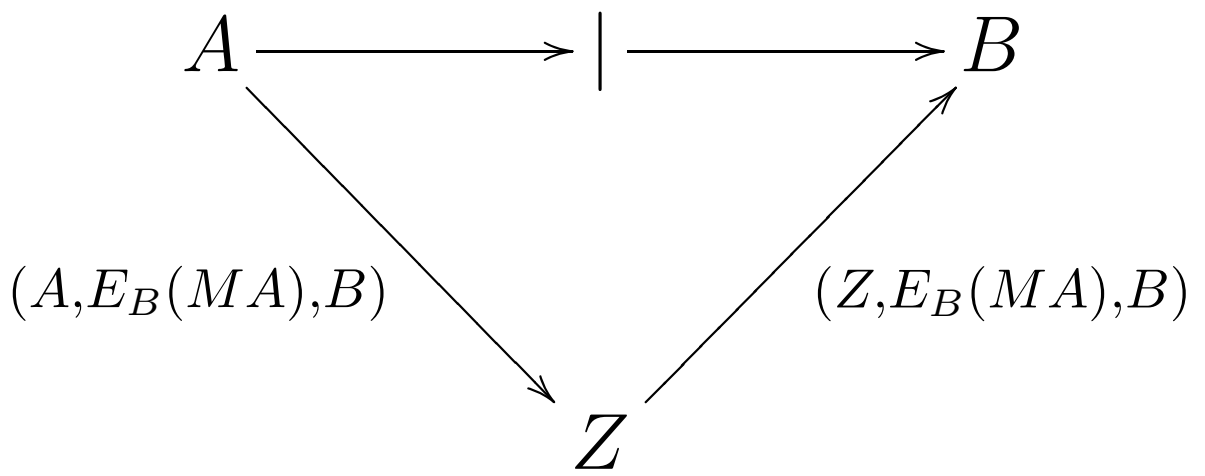
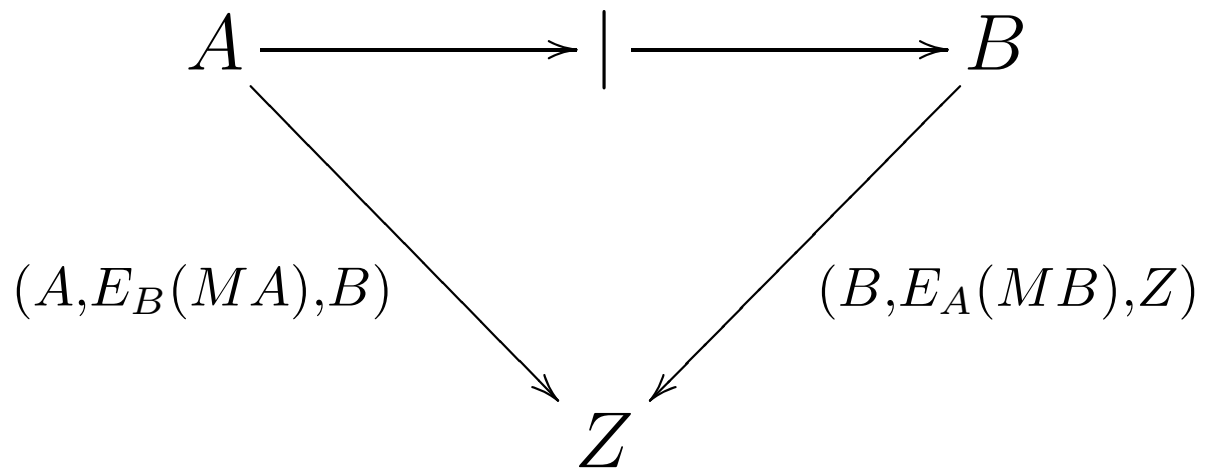


Example 2: Dolev & Yao Model

B sends message $(B, E_A(MB), Z)$ to Z



Intruder Z **cannot** decode $E_A(MB)$ to obtain M

It can be proved that this protocol in secure against arbitrary behaviour of the intruder.

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- Let T be all the information Z has.

Rules : Dolev & Yao Model

Reflexivity

$$\frac{M \in T}{T \vdash M}$$

Encryption

$$\frac{T \vdash K \quad T \vdash M}{T \vdash \{M\}_K}$$

Decryption

$$\frac{T \vdash \{M\}_K \quad T \vdash K}{T \vdash M}$$

Pair – Composition

$$\frac{T \vdash M \quad T \vdash N}{T \vdash (M, N)}$$

Pair – Decomposition

$$\frac{T \vdash (M, N)}{T \vdash M} \qquad \frac{T \vdash (M, N)}{T \vdash N}$$

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2.4 Applying pair composition to 2.1 and 2.3:
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4.3 Pair decomposition to 4.2: $T \vdash E_Z(M)$

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4.5 Applying Decryption rule to 4.3 and 4.4 we obtain: $T \vdash M$

Spi Calculus

- **A Calculus for Cryptographic Protocols: The Spi Calculus**
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- Terms are processes

Spi Calculus

- **A Calculus for Cryptographic Protocols: The Spi Calculus**
  M. Abadi and A. Gordon. Information and Computation , 148(1): 1–70, 1999.

- Similar to the π-Calculus
- Process Algebras
- Terms are processes
- Equivalence Relation: Bisimulation

Language - Spi Calculus

- The language of the Spi-Calculus is very similar to the Pi-Caculus.
- In the standard Pi-Calculus terms are only names.

Terms:

$L, M, N ::=$		Terms
	n	name
	(M,N)	pair
	0	zero
	suc(M)	successor
	x	variable
	$\{M\}_N$	shared-key encryption

Processes - Spi Calculus

$P, Q, R ::=$	Processes
$\bar{M}(N).P$	output
$M(x).P$	input
$P \mid Q$	parallel compos
$(\nu)P$	restriction
$!P$	replication
$[M\ is\ N]P$	match
$\mathbf{0}$	nul
$let\ (x, y) = M\ in\ P$	pair splitting
$case\ M\ of\ 0 :\ suc(x) : Q$	integer case
$case\ L\ of\ \{x\}_N\ in\ P$	shared-key decr

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- Process $case\ L\ of\ \{x\}_N\ in\ P$ attempts to decrypt the term L with the key N. If L is a ciphertext of the form $\{M\}_N$, then the process behaves as $P[M/x]$. Otherwise, the process is stuck.

Example - Spi Calculus

- Example with key establishment

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- Wide Mouthed Frog

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- 3 agents: A, B and S (server)

Example - Spi Calculus

- Example with key establishment
- Wide Mouthed Frog
- 3 agents: A, B and S (server)
- A and B share keys K_{AS} and K_{SB} repectively with server S

Example - Spi Calculus

- Protocol:
- A creates a key K_{AB}
- A send key K_{AB} under encription K_{AS}
- S decript the mesage and send key K_{AB} under encription K_{SB}
- A send mesage M under encription K_{AB}

$$A \xrightarrow{\{K_{AB}\}_{K_{AS}}} S \xrightarrow{\{K_{AB}\}_{K_{SB}}} B$$

Example - Spi Calculus

- **Protocol**

$$A(M) = (\nu K_{AB})(\bar{c}_{AS}\langle\{K_{AB}\}_{K_{AS}}\rangle.(\bar{c}_{AB}\langle\{M\}_{K_{AB}}\rangle))$$
$$S = c_{AS}(x).case\ x\ of\ \{y\}_{K_{AS}}\ in\ \bar{c}_{SB}\langle\{y\}_{K_{SB}}\rangle$$
$$B = c_{SB}(x).case\ x\ of\ \{y\}_{K_{SB}}\ in\ c_{AB}(z).case\ z\ of\ \{w\}_y\ in\ F(w)$$
$$Inst(M) = (\nu K_{AS})(\nu K_{SB})(A(M) \mid S \mid B)$$

- Since all communication is protected by encryption, communication can take place through public channels: c_{AS}, c_{SB} and c_{AB}

Secrecy - Spi Calculus

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- The secrecy property can be stated in terms of equivalences: if $F(M) \simeq F(M'$ for all M and M' , then $Inst(M) \simeq Inst(M')$. This means that if $F(M)$ is indistinguishable from $F(M')$, then the protocol with message M is indistinguishable from the protocol with message M' .

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BAN Logic

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- Set of Rules to manipulate assertions

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- K, ranges over encryption keys.

Sintaxe- BAN Logic

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- $\langle X \rangle_Y$: This represent X combined with the formula Y. In implementations, X is simply concatenated with the password Y.

Logical Postulates - BAN Logic

Message-meaning: rules for interpretation of messages.

For shared keys

$$\frac{P\,\mathbf{believes}\,Q \overset{K}{\leftrightarrow} P, \quad P\,\mathbf{sees}\,\{X\}_K}{P\,\mathbf{believes}\,Q\,\mathbf{said}\,X}$$

For public keys

$$\frac{P\,\mathbf{believes}\,\overset{K}{\mapsto} Q, \quad P\,\mathbf{sees}\,\{X\}_{K^{-1}}}{P\,\mathbf{believes}\,Q\,\mathbf{said}\,X}$$

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Message-meaning: rules for interpretation of messages.

For shared secrets

$$\frac{P\,\mathbf{believes}\,Q \overset{Y}{\rightleftharpoons} P, \quad P\,\mathbf{sees}\,\langle X\rangle_Y}{P\,\mathbf{believes}\,Q\,\mathbf{said}\,X}$$

Logical Postulates - BAN Logic

Jurisdiction:

$$\frac{P \text{ believes } Q \text{ controls } X, \quad P \text{ believes } Q \text{ believes } X}{P \text{ believes } X}$$

Logical Postulates - BAN Logic

Principal sees:

$$\frac{P \,\mathbf{sees}\, (X, Y)}{P \,\mathbf{sees}\, X} \qquad \frac{P \,\mathbf{sees}\, \langle X \rangle_Y}{P \,\mathbf{sees}\, X}$$

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Logical Postulates - BAN Logic

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Logical Postulates - BAN Logic

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$$\frac{P\,\mathbf{believes}\,\forall V_1\ \ldots\ V_n.(Q\,\mathbf{controls}\,X)}{P\,\mathbf{believes}\,Q'\,\mathbf{controls}\,X'}$$

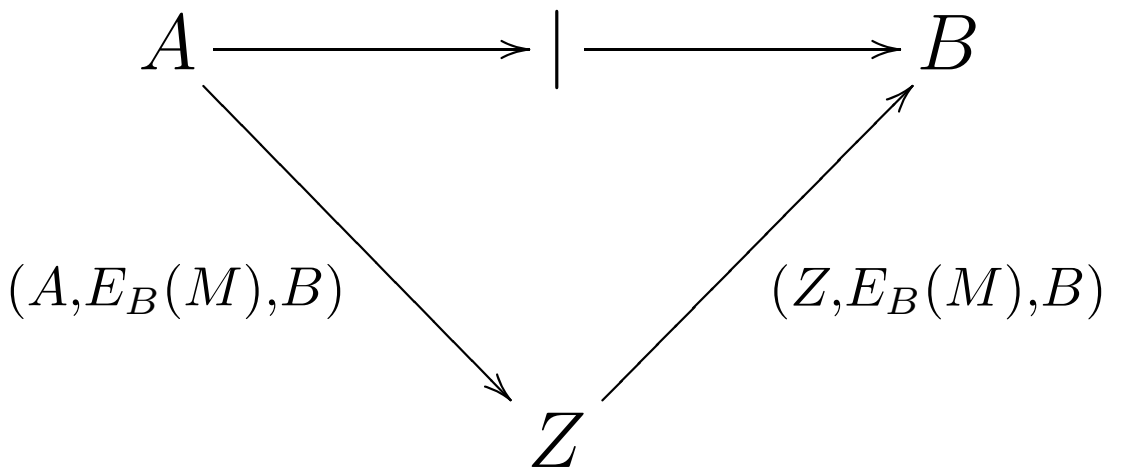
Example 1: Dolev & Yao Model

A sends msg M to B

$$A \longrightarrow (A, E_B(M), B) \longrightarrow B$$

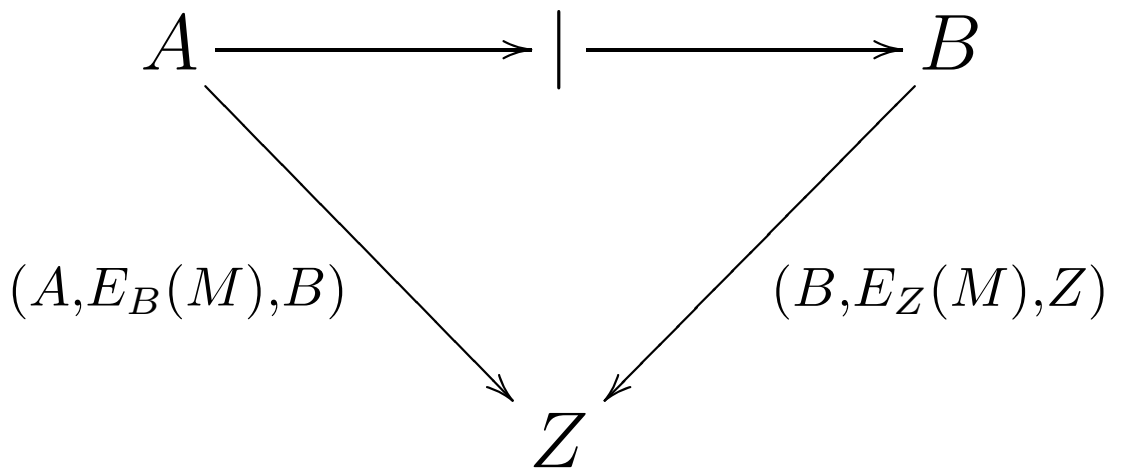
Intruder Z intercepts the message sent from A to B

Intruder Z sends message $(Z, E_B(M), B)$ to B



Example 1: Dolev & Yao Model

B sends message $(B, E_Z(M), Z)$ to Z



Intruder Z decodes $E_Z(M)$ and obtains M

Example - BAN Logic

- $m_1 : A \longrightarrow B : \{m\}_{K_B}$
- $m_2 : Z \longrightarrow B : \{m\}_{K_B}$
- $m_3 : B \longrightarrow Z : \{m\}_{K_Z}$
- B believes $A \overset{K_B}{\leftrightarrow} B$
- Z believes $B \overset{K_Z}{\leftrightarrow} Z$
- $m_1 : Z$ sees $\{m\}_{K_B}$
- $m_2 : B$ sees $\{m\}_{K_B}$
- B sees m $\quad rule\ principal\ sees$
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Dolev/Yao Epistemic Logic

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$$E ::= p \mid k \mid (E_1, E_2) \mid \{E\}_k$$

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- **Formulas**:

$$\varphi ::= e \mid \top \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid K_a\varphi$$

$$\text{where } e \in E, a \in \mathcal{A}$$

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- $\sim_a \subseteq W \times W$ is a reflexive, transitive and symmetric binary relation over W, for each agent $a \in \mathcal{A}$;

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- We call a rooted multi-agent epistemic model $(\mathcal{M}, s)$ an epistemic state.

Semantics - $\mathbf{S5_{DY}}$

Satisfaction: $\mathcal{M}, s \models \varphi$

- $\mathcal{M}, s \models e \; iff \; s \in V(e)$
- $\mathcal{M}, s \models \neg\phi \; iff \; \mathcal{M}, s \not\models \phi$
- $\mathcal{M}, s \models \phi \wedge \psi \; iff \; \mathcal{M}, s \models \phi$ and $\mathcal{M}, s \models \psi$
- $\mathcal{M}, s \models K_a\phi \; iff$ for all $s' \in S : s \sim_a s' \Rightarrow \mathcal{M}, s' \models \phi$

Axiomatization - $\mathrm{S5_{DY}}$

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Inference Rules

M.P. $\varphi, \varphi \to \psi / \psi$ U.G. $\varphi / K_a\varphi$

Soundness - $\mathbf{S5_{DY}}$

Theorem **Soundness**: The following axioms are sound.

- $K_a m \wedge K_a k \rightarrow K_a \{m\}_k \quad (encryption)$
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Common Knowledge - $\mathbf{S5_{DY}}$

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Complexity

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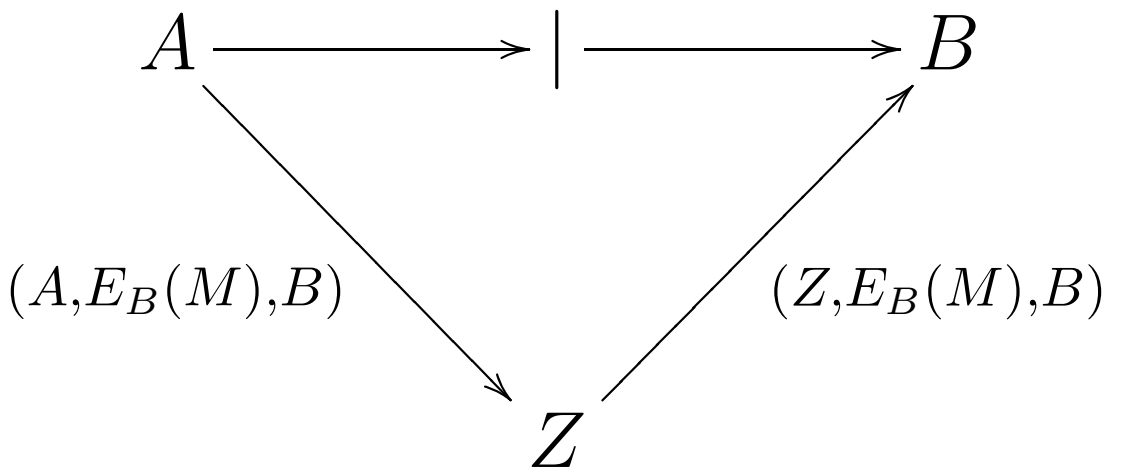
Example 1: Dolev & Yao - Cont.

A sends msg M to B

$$A \longrightarrow (A, E_B(M), B) \longrightarrow B$$

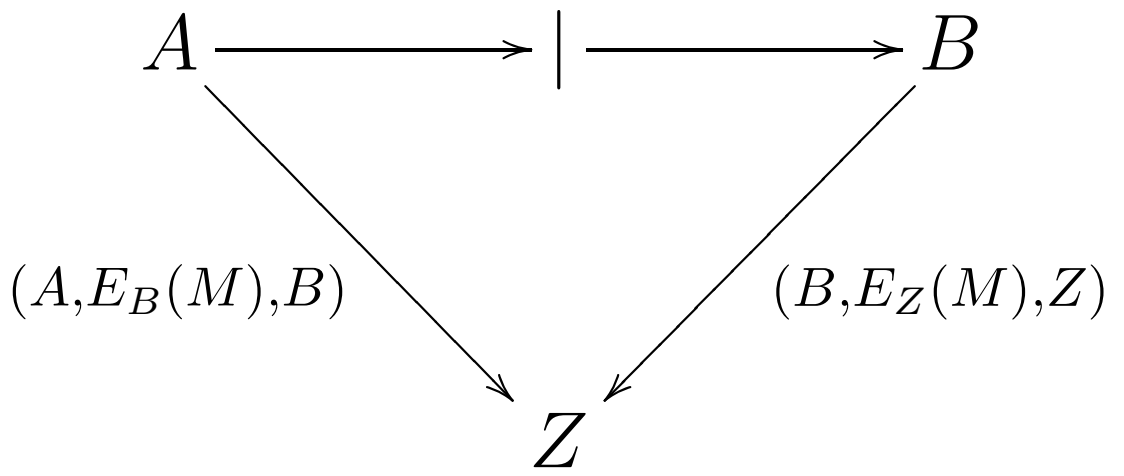
Intruder Z intercepts the message sent from A to B

Intruder Z sends message $(Z, E_B(M), B)$ to B



Example 1: Dolev & Yao - Cont.

B sends message $(B, E_Z(M), Z)$ to Z



Intruder Z decodes $E_Z(M)$ and obtains M

Proving Example 1

- Proving example 1 Dolev & Yao in $\mathbf{S5_{DY}}$

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$$\downarrow send_{AB}(\{m\}_{k_{AB}})$$

$$- - -$$

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$$K_Z m \quad ax.7$$

Intruder Z knows M

Example 2: Dolev & Yao Model

A sends msg MA to B and B replies to the user that is encrypted with the message M and not to the sender

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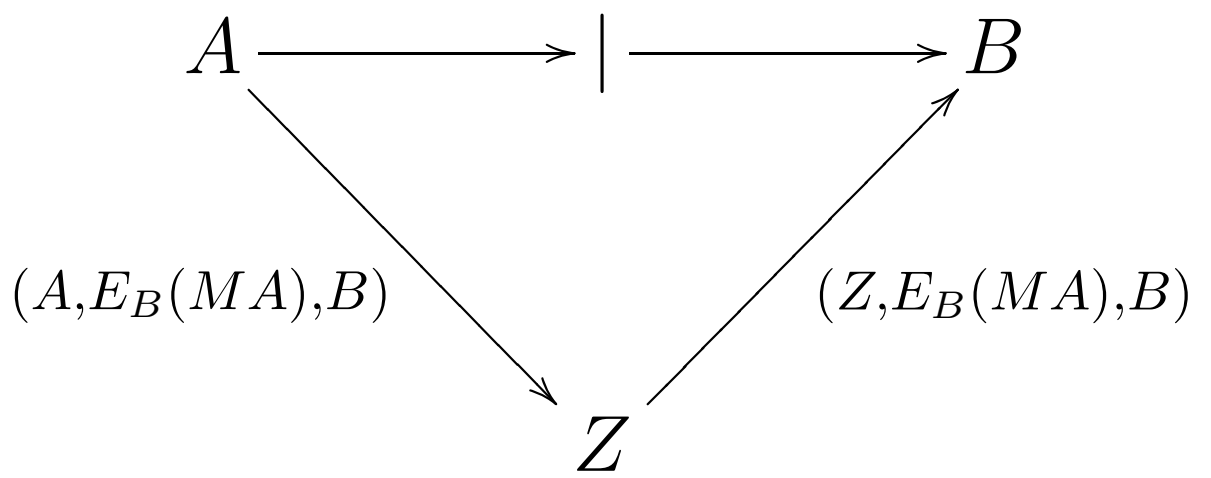
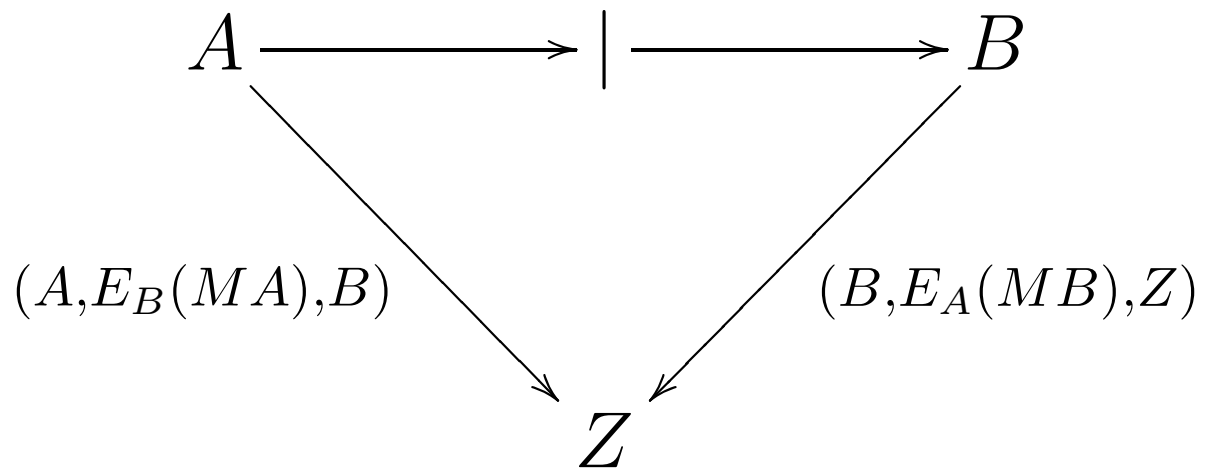


Example 2: Dolev & Yao Model

B sends message $(B, E_A(MB), Z)$ to Z



Intruder Z **cannot** decode $E_A(MB)$ to obtain M

It can be proved that this protocol in secure against arbitrary behaviour of the intruder.

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$$K_B(k_{AB}, m) \quad ax.\ 7.$$

$$K_B m \quad ax.\ 8.$$

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More Examples

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- Action Dolev/Yao Multi-Agent Epistemic Logic $\mathbf{S5^{A}_{DY}}$

  axiom: $K_A m \rightarrow [send_{AB}(m)]K_B m$???????

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- If agent a does know the key K?
- Is there any difference of knowing something and knowing the content of something?

An Extension of the $S5_{DY}$ Logic

- **Message language**:

$$M \ ::= a \mid k \mid (M, M) \mid \{M\}_k$$

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