Open Problems in the II Workshop French-Brazilian in Graphs and Combinatorial Optimization

April 4, 2016

1 Monday's Open Problems

1.1 Victor Campos' Open Problem

1.1.1 Preliminaries

Let G be a simple graph, $R = v_1, v_2, \ldots, v_n$ be an ordering of its vertices and $C(R)$ be the number of colors that the greedy coloring algorithm uses, given the ordering R. We say that R is a connected ordering of the vertices of G if and only if, for each $1 \leq i \leq n$, the subgraph of G induced by the vertices $\{v_1, v_2, \ldots, v_i\}$ is connected. Let $\mathcal{X}_c(G) = \min_{R \in \mathcal{R}_c} C(R)$ and $\Pi_c(G) = \max_{R \in \mathcal{R}_c} C(R)$, where $\mathcal{R}_c$ is the set of all connected orderings of the vertices of G.

Victor Campos proved that $\Pi_c(G) \leq 2$, for bipartite graphs. Also, he proved that $\mathcal{X}(G) \leq \mathcal{X}_c(G) \leq \mathcal{X}(G) + 1$ and that deciding whether $\mathcal{X}_c(G) = \mathcal{X}(G)$ or $\mathcal{X}_c(G) = \mathcal{X}(G) + 1$ is **NP**-complete.

1.1.2 Problem

For some specific graph class $\mathcal{C}$ other than Bipartite Graphs, the problem to decide whether $\mathcal{X}_c(G) = \mathcal{X}(G)$ or $\mathcal{X}_c(G) = \mathcal{X} + 1$ is Polynomial or $\mathbb{NP}$-complete for graphs in $\mathcal{C}$?

1.2 Ueverton Souza's Open Problem

1.2.1 Preliminaries

Let G be a simple graph, $R = v_1, v_2, \ldots, v_n$ be an ordering of its vertices and $C(R)$ be the number of colors that the greedy coloring algorithm uses, given the ordering R. $\Pi(G) = \max_{R \in \mathcal{R}} C(R)$, where $\mathcal{R}$ is the set of all orderings of the vertices of G.

1.2.2 Problem

Is the problem to decide whether $\Pi(G) \geq k$ in **FPT** when it is parameterized by k? (Ueverton's conjecture: No!)

1.3 Julio Araujo's Open Problem

1.3.1 Preliminaries

Let G a simple graph and D be an orientation of the edges of G. Also, let $d_D^-(v)$ be the indegree of v in the orientation D and $\Delta_D^-(G) = \max_{v \in V(G)} d_D^-(v)$. We say that D is a proper orientation of G if and only if, for every pair of neighbors v and v', $d_D^-(v) \neq d_D^-(v')$. Finally, let $\vec{\mathcal{X}}(G) = \min_{D \in \mathcal{D}_p} \Delta_D^-(G)$, where $\mathcal{D}_p$ is the set of all proper orientations of G.

1.3.2 Problems

1. The problem to decide whether $\vec{\mathcal{X}}(G) \leq k$, for a planar graph G and some fixed integer k, polynomial or **NP-** complete?
2. The problem to decide whether $\vec{\mathcal{X}}(G) = w(G) - 1$, for a split graph G, where $w(G)$ is the size of the largest clique of G, polynomial or **NP-** complete?

2 Tuesday's Open Problems

2.1 Rudini Sampaio's Open Problem

2.1.1 Preliminaries

A split graph is a graph in which the vertices can be partitioned into a clique and an independent set. Let G be a split graph. Deciding $\mathcal{X}(G)$ of the Split Graphs is easy. Deciding the $\mathcal{X}_h(G)$ (chromatic harmonious number) is NP-Hard. A star coloring $\mathcal{X}_{ST}(G)$ of a graph G is a proper vertex coloring in which every path of four vertices uses at least three distinct colors.

2.1.2 Problem

For a split graph G with partition (C,S) such that $|C| = k$, is polynomial decide whether $\mathcal{X}_{ST}(G) = k$ or $\mathcal{X}_{ST}(G) = k + 1$?

2.2 Frederic Havet's Open Problem

2.2.1 Preliminaries

A subdivision of a digraph F, also called an F-subdivision, is a digraph obtained from F by replacing each arc ab of F by a directed (a,b)-path.

2.2.2 Problem

Consider the following problem for a fixed digraph F and a fixed family of digraph $\mathcal{F}$.

Input: A digraph D

Questions: Does D contain a subdivision of F as a subgraph? Does D contain a subdivision of F as subgraph for some F $\in \mathcal{F}$?

2.3 Nicolas Trotignon's Open Problem

2.3.1 Preliminaries

Analogous to Frederic Havet's problem.

2.3.2 Problems

Consider the following problem for a fixed graph F.

Input: A graph G

Questions: Does G contain an induced subdivision of F as a subgraph?

3 Wednesday's Open Problems

3.1 Rudini Sampaio's Open Problem

3.1.1 Preliminaries

A *locally identifying coloring* (lid-coloring) of a graph is a proper coloring such that the sets of colors appearing in the closed neighborhoods of any pair of adjacent vertices having distinct neighborhoods are distinct. Let $\chi_{\text{lid}}(G)$, *lid-chromatic number*, be the minimum number of colors used in a locally identifying vertex-coloring of G. Rudini proved that lid-chromatic number is $n^{1/2-\varepsilon}$-inapproximable in polynomial-time, unless P=NP, for each $\varepsilon > 0$.

3.1.2 Problems

Is lid-chromatic number $n^{1-\varepsilon}$-inapproximable in polynomial-time, unless P=NP, for each $\varepsilon > 0$?

4 Thursday's Open Problems

4.1 Claudia Linhares's Open Problem

4.1.1 Preliminaries

Given a directed graph D the *acyclic chromatic number* of D denoted by $\chi_A(D)$ is the minimum positive integer k such that the vertices of D can be partitioned into sets $V_1, \dots, V_k$ such that each set V_i induces a directed acyclic graph(DAG) in D.

4.1.2 Problems

1. Is it true that for any graph G such that $\chi(G) \geq 100$ there exists an orientation $\vec{G}$ of G such that $\chi_A(\vec{G}) \leq 3$?

2. Is it true that for any graph G such that $\chi(G) \geq 100$ for any induced subgraph H of G there exists an orientation $\vec{H}$ of H such that $\chi_A(\vec{H}) \leq 3$?

4.2 Nicolas Nisse's Open Problem

4.2.1 Preliminaries

Let $G_{(n,\infty)}$ be the grid graph with n rows and an infinity number of columns numbered $0, 1, 2, 3 \ldots$. There are two players(B and F). The F player starts positioning k defenders on the vertices of $G_{(n,\infty)}$ and, after that, the B player places one attacker in one of the vertices of column 0.

At each turn the player F may move his defenders to an adjacent vertex and then the player B can move his attacker to a vertex with distance at most 2 from its current position. The goal for the player B is to increase indefinitely the column where the attacker is positioned, never occupying the same vertex of a defender. The goal for player F is to position a defender in the same vertex of the attacker.

Let $g_2(n)$ denote the minimum k such that the player F can always win using k defenders on $G_{(n,\infty)}$.

4.2.2 Problems

Is there a $\epsilon > 0$, such that for all $n > 0$, $g_2(n) \leq n^{1-\epsilon}$?