On the Minimal Number of Convex Sets on the P_3 and P_3^* convexities*

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***Abstract*—Let $G = (V, E)$ be a finite graph. A convexity on $V(G)$ is a family $\mathcal{C}$ of subsets $V(G)$ such that $\emptyset, V(G) \in \mathcal{C}$ and $\mathcal{C}$ is closed under intersections. For the geodetic and monophonic convexities, Brown and Oellermann studied the graphs such that the convex sets were only the unavoidable [Graphs with a Minimal Number of Convex Sets, Graphs and Combinatorics 30 (2014) 1383–1397]. Following the same line of research, we study graphs with a minimum number of convex sets on the P_3 and P_3^* convexities.**

I. Introduction

Let V be a finite set. A convexity is an ordered pair $(V, \mathcal{C})$ where V is a ground set and $\mathcal{C}$ is a family of subsets of V such that $\emptyset, V(G) \in \mathcal{C}$ and $\mathcal{C}$ is closed under intersections. For graphs convexities the ground set is usually the vertex set. Graph convexities have been studied since the 70's, for finite graphs.

Usually graph convexities are considered with respect to a family of paths. In this case, a set is convex if it is closed under the family of paths being considered. Among the most studied convexities, we can mention the geodetic convexity [1], in which the convex sets are closed under shortest paths, the monophonic convexity [2], in which the convex sets are closed under induced paths, the P_3 convexity [3], in which the convex sets are closed under paths of order three, and the P_3^* convexity [4], in which the convex sets are closed under induced paths of order three.

In this work we consider the concept of minimal graphs with respect to a graph convexity. A graph G is said minimal with respect to a convexity if the only sets of G are the empty set, $V(G)$, all singletons and all edges. Brown and Oellermann [5] have established several necessary conditions for a geodetic minimal graph. In the case of monophonic convexity, they gave a characterization of geodetic minimal graphs in terms of structural properties.

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Following the same line of research, we consider minimal graphs of P_3 and P_3^* convexities. We discuss similarities of these cases with the previously studied cases, and present some partial results on the quest of characterizing such graphs. In particular, at least in the case of the P_3 convexity, it is possible to characterize minimal graphs in terms of structural properties and local convexity properties.

References

[1] M. Farber, R.E. Jamison, *Convexity in graphs and hypergraphs*, SIAM J. Algebraic Discrete Methods 7 (1986) 433–444.

[2] P. Duchet, *Convex sets in graphs. II: minimal path convexity*, J. Comb. Theory, Ser. B 44 (1988) 307–316.

[3] R. Barbosa, E. M. Coelho, M. C. Dourado, D. Rautenbach, J. L. Szwarcfiter, *On the Carathéodory number for the convexity of paths of order three.*, SIAM Journal on Discrete Mathematics 26.3 (2012) 929-939.

[4] R. Araújo; R. M. Sampaio, V. F. dos Santos, J. L. Szwarcfiter, *The convexity of induced paths of order three and applications: complexity aspects*, submitted.

[5] J. Brown, O. R. Oellermann, *Graphs with a Minimal Number of Convex Sets*, Graphs and Combinatorics 30 (2014) 1383–1397.