Stable project problem's models

Bertrand Le Cun

Laboratoire PRiSM, UniversitÃ© de Versailles-Saint-Quentin

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Max Stable Set Problem

$$\begin{array}{lll} \max & \sum_{i=1}^{n} x_i & \\ \text{s.t.} & x_i + x_j \leq 1 & if\ (i,j) \in E \\ & x_i \in \{0,1\} & \forall i \in [1..n] \end{array}$$

$x_i = 1$ if vertex i is in the stable, 0 otherwise.

Coloring Problem

$$
\begin{array}{lll}
\text{Min} & \sum_{i=1}^{n} x_i^u & \\
\text{s.t.} & x_i^u + x_j^u \leq x_u^u & if\ (i,j) \in E, (u,i)(u,j) \notin E \\
& \sum_{u=1, u_i \notin E} x_i^u = 1 & \forall i, \in [1..n] \\
& x_i^u \in \{0,1\} & \forall 1 \leq u \leq i \leq n
\end{array}
$$

$x_i^u = 1$ if vertex i is in the stable set whose representative i u, 0 otherwise.

Max stable set weighted by subgraph

$$
\begin{array}{lll}
\text{Max} & \sum_{h=subgraph} w_h x_h & \\
\text{s.t.} & x_i + x_j \leq 1 & if\ (i,j) \in E \\
 & y_h \leq \sum_{i \in H} x_i & \forall h \\
 & x_i \in \{0,1\} & y_h \in 0,1
\end{array}
$$

$x_i = 1$ if vertex i is in the stable set
$y_h = 1$ if h is touched, 0 otherwise.

K-Partit

Given G(V,E) find a k-partit induced subgraph as large as possible

$$
\begin{array}{lll}
\text{Max} & \sum_i \sum_u x_i^u & \\
\text{s.t.} & x_i^u + x_j^u \leq x_u^u & (u,i),(u,j) \notin E, (i,j) \in E \\
 & \sum_{u=1} x_u^u = k & \forall i, \in [1..n] \\
 & x_i^u \in \{0,1\} & \forall u, i
\end{array}
$$

Weighted Coloring problem

Given G(V,E) find a k-partit induced subgraph as large as possible

$$
\begin{array}{lll}
\text{Max} & \sum_i \sum_u x_i^u & \\
\text{s.t.} & x_i^u + x_j^u \leq x_u^u & (u,i),(u,j) \notin E, (i,j) \in E \\
 & \sum_{u=1} x_u^u = k & \forall i, \in [1..n] \\
 & x_i^u \in \{0,1\} & \forall u, i
\end{array}
$$

Weight Coloring Problem

$$
\begin{array}{lll}
\text{Min} & \sum_{i=1}^{n} w_u x_i^u & \\
\text{s.t.} & x_i^u + x_j^u \leq x_u^u & if\ (i,j) \in E, (u,i), (u,j) \notin E \\
 & \sum_{u=1, u_i \notin E} x_i^u = 1 & \forall i, \in [1..n] \\
 & x_i^u \in \{0,1\} & \forall 1 \leq u \leq i \leq n
\end{array}
$$

$x_i^u = 1$ if vertex i is in the stable set whose representative i u, 0 otherwise.
vertices are ordered in decreasing order of w_u