STIC AmSud Project
Solving Combinatorial Optimization Problems with Stable Sets Constraints

Universidade Federal do Ceará, Brazil
Universidad de Concepción, Chile
Universidad de Santiago de Chile, Chile
Unversidad Nacional de General Sarmiento, Argentine
Université de Versailles Saint-Quentin, France
Université d'Avignon et des Pays de Vaucluse, France

Support:

CAPES, Brasil   CONICYT, Chile   MINCYT, Argentine
MAEE and CNRS, France

Clarifications/Reminders

Project Duration

1st year: February, 2013 - December, 2013

2nd year: January, 2014 - December, 2014 (if renewed)

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Partial Report

- Due date: October 1st, 2013
- Activities of the first year/ Planning for the 2nd year
- Current Status of the project (financial report, potential academic and institutional gains)
- Basis for the renewal

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Final Report

- Due date: 30 days after the end of the project.
- Developped activities
- Attained goals and results (joint publications, trainning)

Clarifications/Reminders

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1st year: February, 2013 - December, 2013
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Accountability Report

- Due date: 30 days after the end of each year
- To be sent separately to each agency (please check)
- Please check the required supporting documentation (receipts, tickets etc)

Clarifications/Reminders

Project Duration

1st year: February, 2013 - December, 2013
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Project webpage

www.

Missions for the 1st year

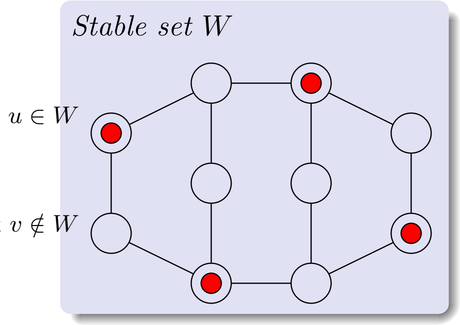
Complete the table

Mission	Name	num. days	Period
Brasil → Argentine	Manoel	13	
Brasil → Argentine	Ricardo	13	
Brasil → França	Shirley C.	15	
Brasil → França	Victor C.	15	
França → Brasil	Boris	12	
Chile → França	Victor P.	17	
Chile → Argentine	Rodrigo	10	
Argentine → Chile		15	

Project Objectives

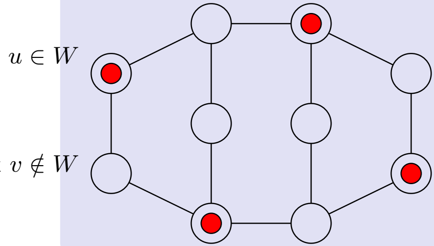
- Strengthen and extend a collaboration network
- Train and form young researchers, specially Ph.D. Students
- Strengthen doctoral programs of the involved institutions
- Increase the scientific production (publications) of the involved teams
- Study combinatorial problems modelled by stable sets constraints
- Perform theoretical studies of the polytope induced by the stable sets constraints
- Design efficient algorithms for solving the studied problems

Combinatorial Problems with Stable Sets Constraints



Combinatorial Problems with Stable Sets Constraints

Stable set W



Modelling $\mathcal{W}$

$$x[u] \in \{0, 1\}, \quad u \in V(G)$$
$$x[u] + x[v] \leq 1, \quad uv \in E(G)$$

Stable Set Polytope

$STAB(G)$ = convex hull of vectors x describing $\mathcal{W}$

Applications

- Timetabling: classes given by the same teacher
- Scheduling: tasks executed at the same time
- frequency assignment for cell phones: links operating on the same frequency
- communication scheduling on wireless network: links communicating simultaneously without interferene
- ...

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- Haplotyping inference: genotypes explained by the same haplotying

Disjoint Stable Sets Problems

Finding a family of stable sets under certain constraints, such as

- cardinality constraints
- capacity constraints
- covering constraints
- packing constraints

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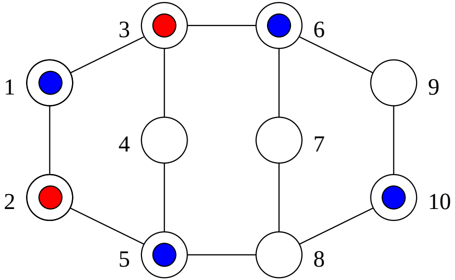
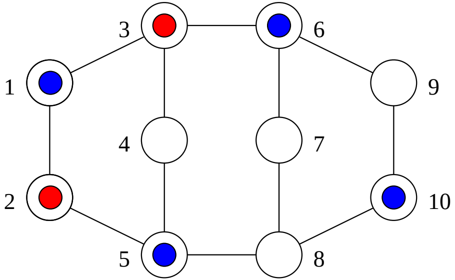
For the ease of the explanation, let us assume the family $\mathcal{W}$ we want is composed by disjoint stable sets

Modelling $\mathcal{W}$ by Representatives



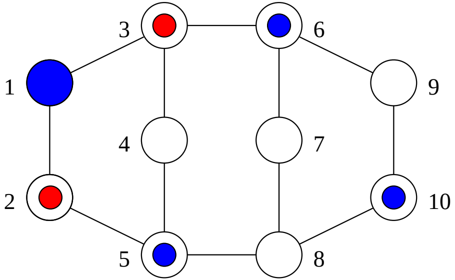
Modelling $\mathcal{W}$ by Representatives

- Choose a representative for each nonempty stable set of G



Modelling $\mathcal{W}$ by Representatives

- Choose a representative for each nonempty stable set of G
- One simple criterion is to choose the smallest vertex in the stable set according to a given order of the vertices [C, Campos, C 2008].



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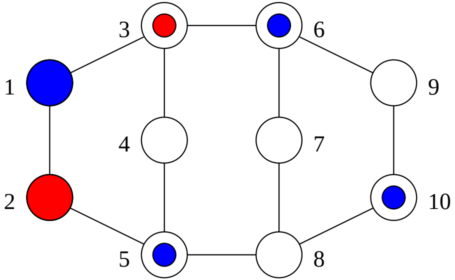
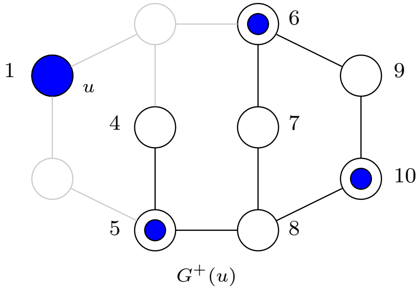


Notation for the Representatives



Anti-neighborhood

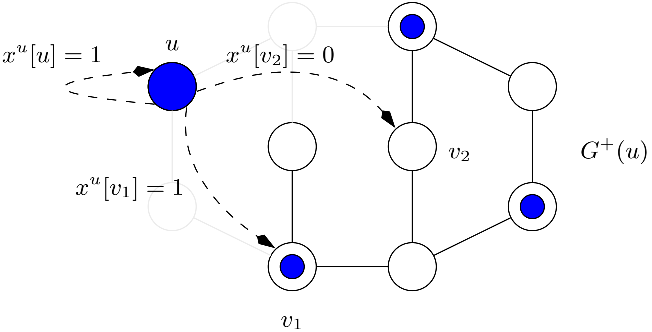
$$\bar{N}^+(u) = \{v > u \mid uv \notin E\}$$

Subgraph induced by anti-neighbors

$$G^+[u] = G[\bar{N}^+[u] = \bar{N}^+(u) \cup \{u\}]$$

Stable sets represented by u
Stable sets of $G^+(u)$ containing u itself

Variables



For each $u \in V(G)$:

$$x^u : \text{ binary vector indexed by } \bar{N}^+[u] = \bar{N}^+(u) \cup u$$
$$x^u[u] \in \{0,1\} : u \text{ is a representative}$$
$$x^u[v] \in \{0,1\} : u \text{ represents } v \in \bar{N}^+(u)$$

Formulations by Representatives

Stable set constraints

For each $u \in V$:

$$x^u[v] + x^u[w] \leq x^u[u], \quad v, w \in \bar{N}^+(u), vw \in E \tag{1}$$

$$x^u[v] \leq x^u[u], \quad v \in \bar{N}^+(u) \tag{2}$$

$$x^u[v] \in \{0, 1\}, \quad v \in \bar{N}^+[u] \tag{3}$$

$+$

Specific Constraints

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Parameterized Stable Set Polytope

- $STAB_r(G^+[u])$: convex hull of (1)-(3)

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Parameterized Stable Set Polytope

- $STAB_r(G^+[u])$: convex hull of (1)-(3)
- $STAB_r(G^+[u])$ can be described as $STAB(G^+(u))$

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Specific Constraints

Parameterized Stable Set Polytope

- $STAB_r(G^+[u])$: convex hull of (1)-(3)
- $STAB_r(G^+[u])$ can be described as $STAB(G^+(u))$
- How do the additional constraints affect the problem: polyhedral structure, solution method ?

General Disjoint Stable Sets Problem

$$
\begin{pmatrix}
-x^{v_1}[v_1] & & & x^{v_1}[\bar{N}^+(v_1)] & & \\
& \ddots & & & \ddots & \\
& & -x^{v_n}[v_n] & & & x^{v_n}[\bar{N}^+(v_n)] \\
a^{v_1}x[v_1] & \cdots & a^{v_n}x[v_n] & & & \\
c_{v_1}x^{v_1}[v_1] & \cdots & c_{v_n}x^{v_n}[v_n] & c^{v_1}x^{v_1}[\bar{N}^+(v_1)] & \cdots & c^{v_n}x^{v_n}[\bar{N}^+(v_n)]
\end{pmatrix}
\leq
\begin{pmatrix} 0 \\ \vdots \\ 0 \\ b \\ d \end{pmatrix}
$$

Problems

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- Solution methods

Thank you

Philippe's turn !