On edge b-colorings

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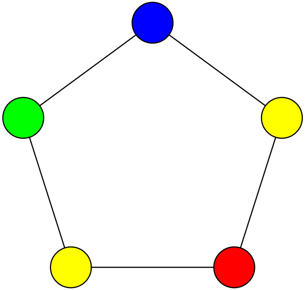
April 2nd, 2013

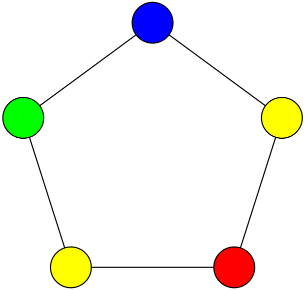
Vertex Coloring



Vertex Coloring

- one color for each vertex
- adjacent vertices get different colors

Vertex coloring problem



Definition: chromatic number

- $\chi(G) =$ min number of colors in a vertex coloring of G

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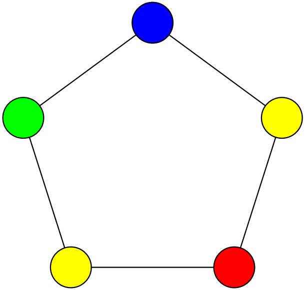

Definition: vertex coloring problem

- $\chi(G) =$ min number of colors in a vertex coloring of G
- Vertex coloring problem: find $\chi(G)$

Some considerations

Dificulties:

- Decision version is NP-complete

R. Karp
Reducibility among combinatorial problems.
Complexity of Computations, 1972.

Some considerations

Dificulties:

- Decision version is NP-complete
- Impossible to approximate by a factor of $n^{1-\epsilon}$ unless P=NP

C. Lund e M. Yannakakis
On the hardness of approximating minimization problems.
Journal of the ACM, 1994.

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- Study polinomial coloring algorithms

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- Not guaranteed to find optimal solutions

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  - Greedy algorithm
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- Not guaranteed to find optimal solutions
- So we study the worst case of these algorithms:
  - The worst case of the greedy algorithm is the *Grundy number* ($\Gamma(G)$)
  - The worst case of the b-coloring algorithm is the *b-chromatic number* ($b(G)$)

Other considerations

It's hard!

- finding $\Gamma(G)$ or $b(G)$ is NP-hard

Other considerations

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- finding $\Gamma(G)$ or $b(G)$ is NP-hard
  - even if G is bipartite

F. Havet e L. Sampaio
On the grundy number of a graph.
IPEC, 2010.

J. Kratochvil, Z. Tuza e M. Voigt
On the b-chromatic number of graphs.
LNCS, 2002.

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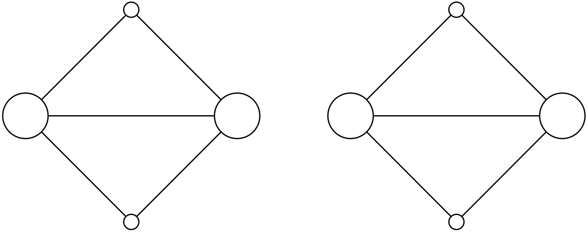
  - line graphs

b-coloring algorithm



Algorithm:

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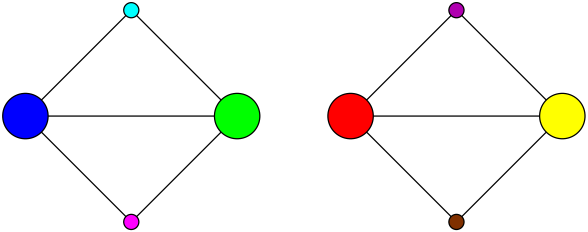

Algorithm:

1. Give different colors to all vertices of G.

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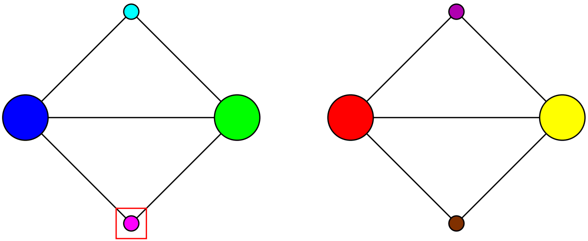
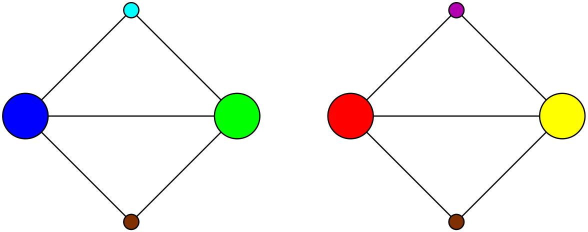


Algorithm:

1. Give different colors to all vertices of G.
2. Choose a color class we can recolor.

b-coloring algorithm



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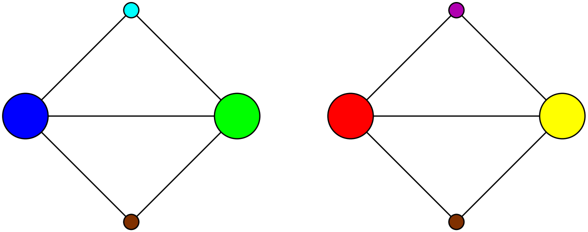

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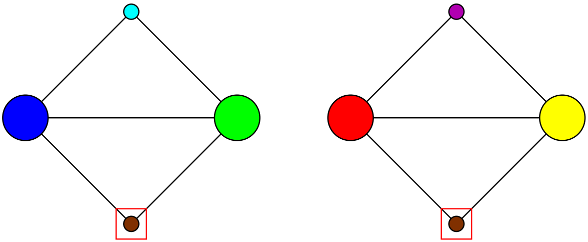

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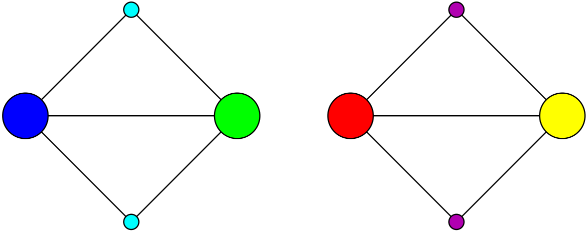

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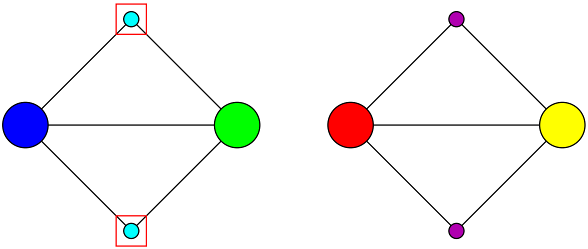

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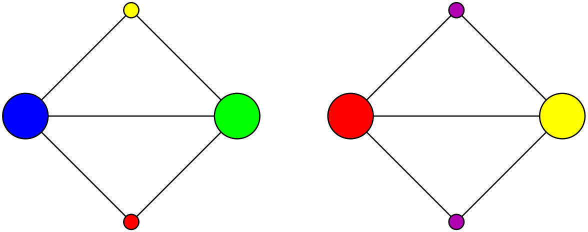

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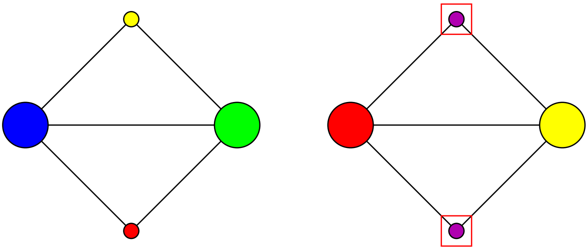

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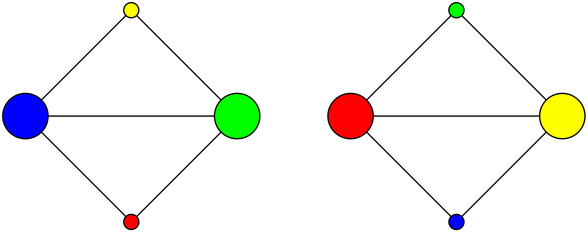

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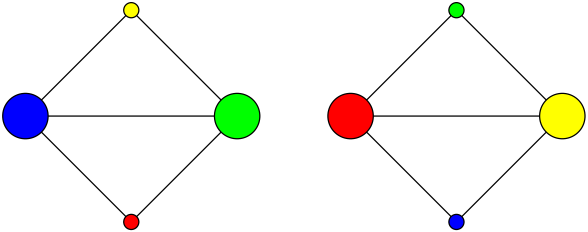

Definitions:

- b-vertex: at least one neighbor of each color other than its own.

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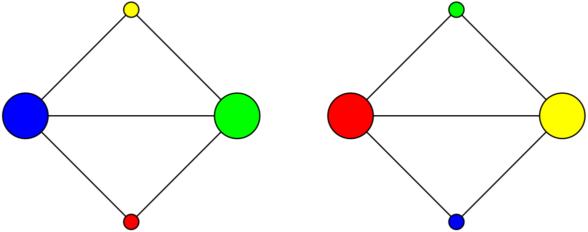

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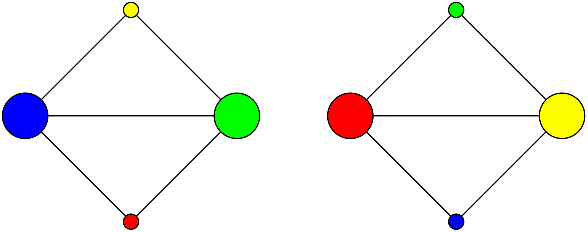

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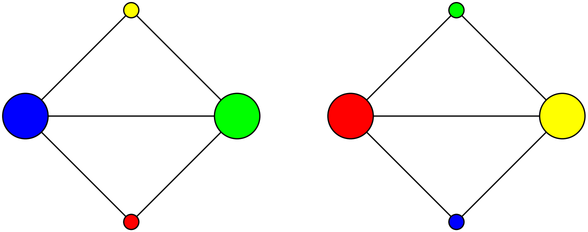
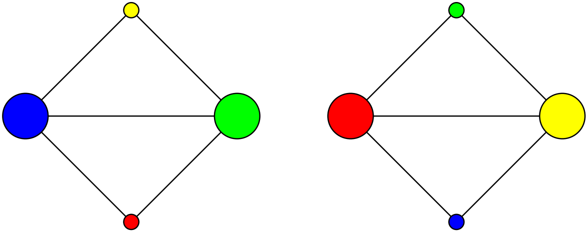
- $b(G) =$ max number of colors in a b-coloring of G.

Upper bound



Has 4 vertices of degree 3.

Upper bound



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Definition: *m*-degree

$$m(G) = \max\{k \mid G \text{ has } k \text{ vertices with degree } \geq k-1\}$$

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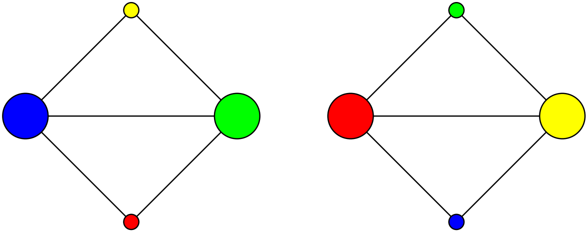

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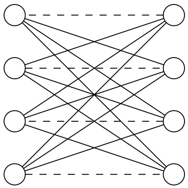
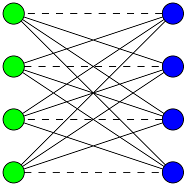
$$\chi(G) \leq b(G) \leq m(G)$$

b-continuity



$K_{4,4}$ minus a perfect matching.

b-continuity



$K_{4,4}$ minus a perfect matching.

- Has b-coloring with 2 colors.

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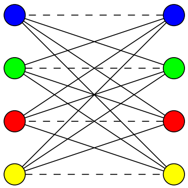

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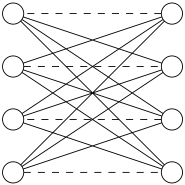

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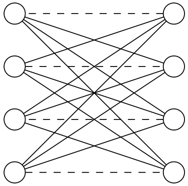

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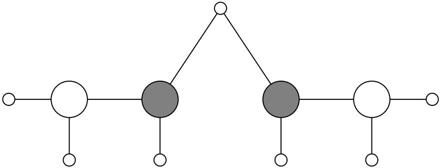
G is *b-continuous* if if has a b-coloring with k colors for every $k \in \{\chi(G), ..., b(G)\}$

b-chromatic number of trees



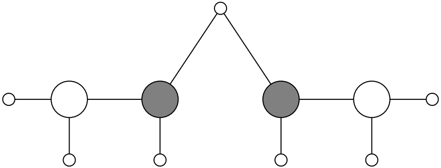
Theorem [IM]

If T is a tree, then

- $b(T) \in \{m(T) - 1, m(T)\}$

R. Irving e D. Manlove
The b-chromatic number of a graph.
Discrete Applied Mathematics 91, 1999.

b-chromatic number of trees



Theorem [IM]

If T is a tree, then

- $b(T) \in \{m(T) - 1, m(T)\}$
- There exists a polinomial time algorithm to decide $b(T)$.

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Our results

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- Instance: Graph G
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Theorem [CLMSSS]

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Theorem [CLMSSS]

- Edge b-chromatic problem is NP-complete.
- It remains NP-complete even if G is a comparability graph or C_k-free graph, for $k \geq 4$.

Other questions of interest

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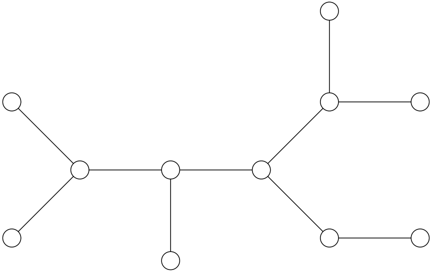

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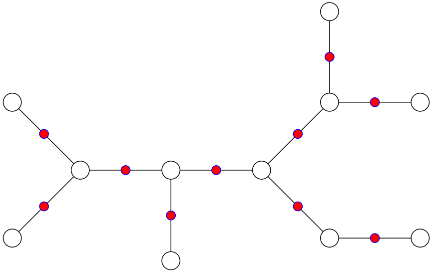

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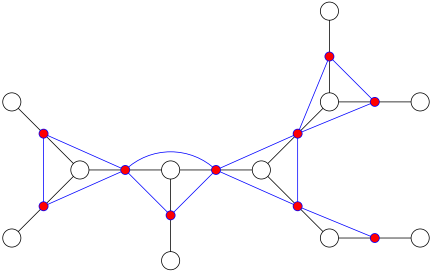

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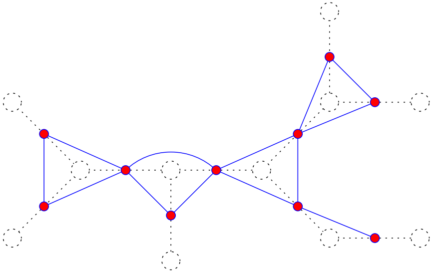

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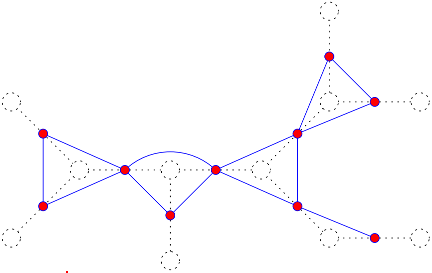

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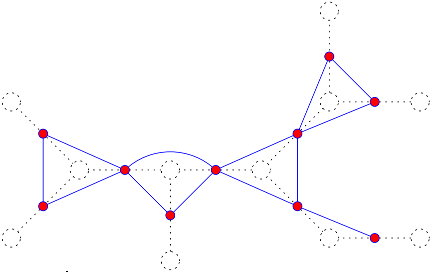

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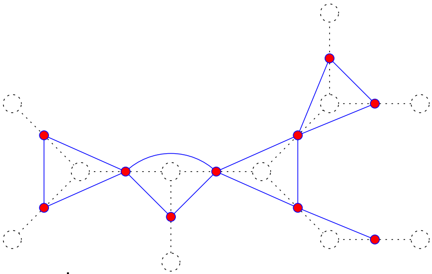
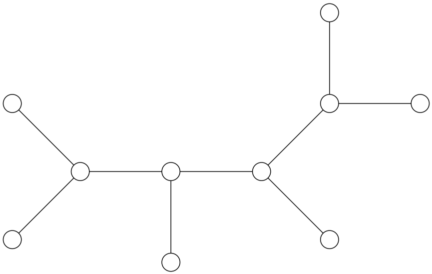


- Special block graphs.
- b-coloring block graphs are hard! (in the sense that it's unknown)

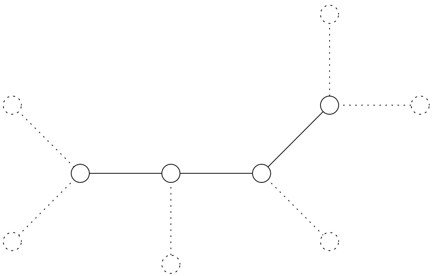
Caterpillar trees



Caterpillar T

Deleting all leaves of T produces a path.

Caterpillar trees



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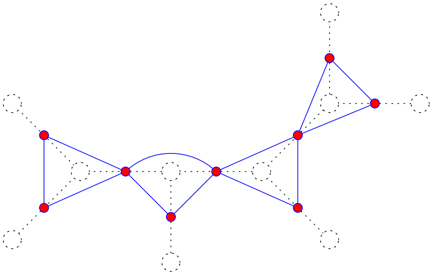

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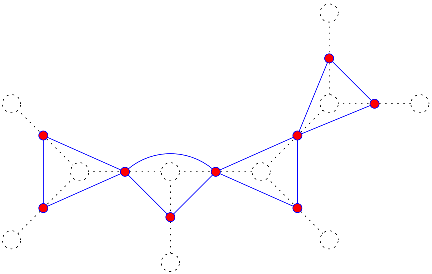

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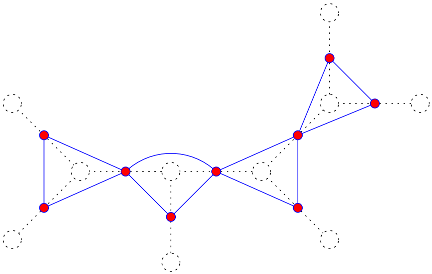

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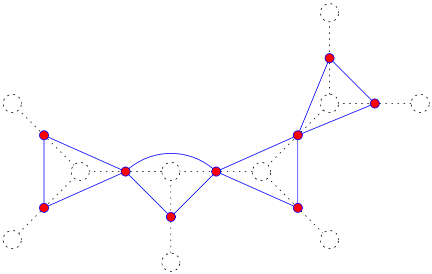
- Line graphs of trees are chordal.
- Chordal graphs are perfect.

Caterpillar trees



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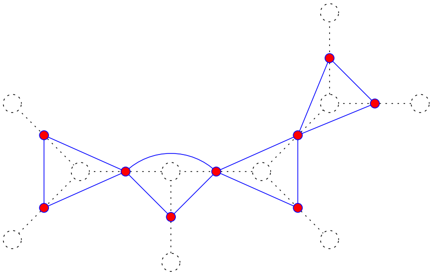

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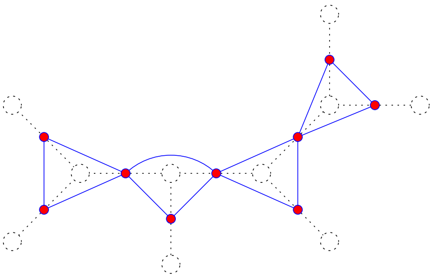

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- $\chi = \omega =$ size of largest clique.
- From now on, let G be a line graph of a caterpillar.

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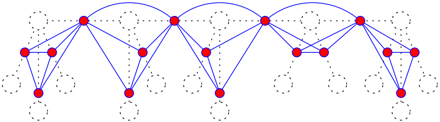
Corollary [CLMSSS]

$b(G) \in \{m(G) - 1, m(G)\}$.

Deciding the value of $b(G)$

- Suppose $\chi(G) < m(G)$.



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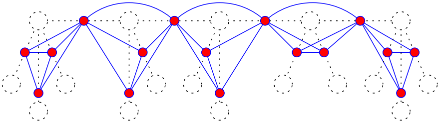

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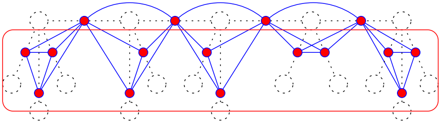

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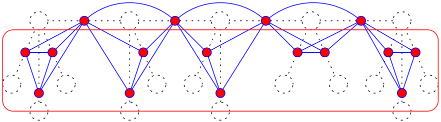

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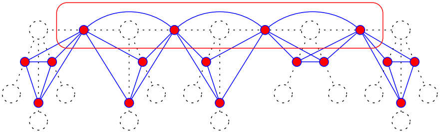
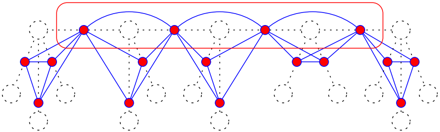
- Only possible b-vertices are in the *central path*.



An interesting problem

Central path coloring problem

- Instance: Graph G and subset $W = \{w_1, \ldots, w_k\}$ of vertices in the central path
- Question: Is there a b-coloring of G with $m(G)$ colors such that the vertices of W are b-vertices of different colors?



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Theorem [CLMSSS]

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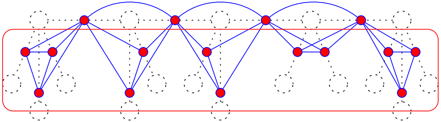
Solving the CENTRAL PATH COLORING PROBLEM



- Give colors to cliques.

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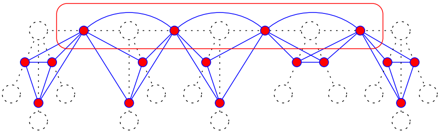

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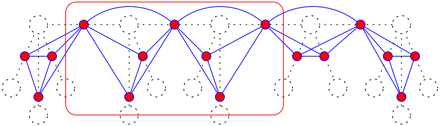

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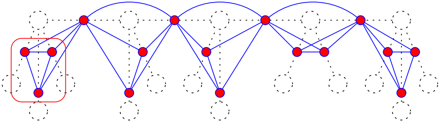

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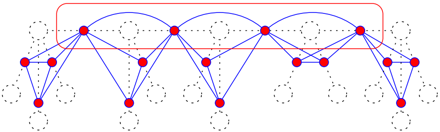

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Let this be the Central path polytope.

Results on the Central path polytope

Theorem [CLMSSS]

If the Central path polytope has an integer solution, then the Central path coloring problem is true.

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Corollary [CLMSSS]

There is a polynomial time algorithm to decide the Central path coloring problem.

Results on the CENTRAL PATH POLYTOPE

Theorem [CLMSSS]

If the CENTRAL PATH POLYTOPE has an integer solution, then the CENTRAL PATH COLORING PROBLEM is true.

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Corollary [CLMSSS]

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Corollary [CLMSSS]

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Open problems

- Obtain a combinatorial algorithm to solve the CENTRAL PATH COLORING PROBLEM.

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  - Use the primal-dual method?

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- Generalize combinatorial algorithm for other classes of trees.

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- Profit!

The End

Thank You!!