Combinatorial ALgorithms Applied to Network problems using Graphs and Optimization (CALANGO)

ParGO, Federal University of Ceará
COATI Project, I3S Laboratory (UMR 7271)
G-SCOP Laboratory, Grenoble INP, CNRS and Université Grenoble Alpes

Abstract

The goal of this project is to continue a long-standing collaboration between the COATI project and the ParGO research group, now reinforced with the AlGCo project and the team of Combinatorial Optimization of G-SCOP Laboratory. These teams share common research interests: from mathematically modelling Network problems using Graph Theory and/or Operations Research to solving such problems by designing, analyzing, implementing and testing efficient algorithms (or by proving that such algorithms do not exist under some complexity hypothesis), going as well through a fine-grained complexity analysis via the lens of the Parameterized Complexity framework. Moreover, the teams have a long history of sharing master and PhD students, as well as visiting professors. Thus, this proposal also aims at providing a better formation for the students of these groups, by providing them the opportunity of having long-term visits, as well as providing the oportunity to young researchers of doing postdocs in other research groups.

Keywords: Network problems, Combinatorial Optimization, Algorithms, Computational Complexity, Parameterized Complexity.

1 Description of the partners

1.1 Brazilian partner

ParGO, Federal University of Ceará
`http://www.lia.ufc.br/~pargo/`

The Federal University of Ceará is the most important and well-known university in the State of Ceará, and one of the biggest universities in Brazil. The ParGO (Parallelism, Graphs and Optimisation) team is linked to the PhD Program in Computer Science at the Federal University of Ceará. Our research group was founded in 1999, initially with 03 researchers - Ricardo Corrêa, Cláudia Linhares Sales and Manoel Campêlo. Since then it has expanded its activities in the university and in the state. Currently, the the group is formed by 15 researchers from three different departments: Computer Science, Statistics and Applied Mathematics, and Mathematics. With integrated acting in areas that define its name, the ParGO researchers have relevant scientific production and have a prominent role in supervision of masters and PhD students.

The group's research focus is on discrete mathematics problems and their applications. The expertises of its members are complementary and enable approaching different aspects of the considered problems. This involves computational complexity studies, design and analysis of algorithms, modeling via graphs or mathematical programming, problem decomposition techniques, resolution using high performance computing. On the other hand the problems studied by the ParGO team are usually motivated by and have applications in telecommunication networks, social networks, cryptography, etc.

Participants

Professors:

- **Cláudia Linhares Sales** (coordinator, Departamento de Computação, UFC)
- Júlio C. S. Araújo (Departamento de Matemática, UFC)
- Rafael C. de Andrade (Departamento de Estatística, UFC)
- Fabrício S. Benevides (Departamento de Matemática, UFC)
- Manoel B. Campêlo Neto (Departamento de Estatística e Matemática Aplicada, UFC)
- Victor A. Campos (Departamento de Computação, UFC)
- Ana K. Maia (Departamento de Computação, UFC)
- Rudini M. Sampaio (Departamento de Computação, UFC)
- Ana S. Silva (Departamento de Matemática, UFC)
- Ronan P. Soares (Departamento de Estatística, UFC)

PhD Students:

- Eurinardo Rodrigues Costa
- Jefferson Lourenço Gurguri
- Luiz Alberto do Carmo Viana
- Mardson da Silva Ferreira
- Paulo Henrique Macêdo de Araújo
- Rafael Teixeira de Araújo
- Raul Wayne Teixeira Lopes
- Rennan Ferreira Dantas
- Tatiane Fernandes Figueiredo

MSc Students:

- Allen Roossim Passos Ibiapina
- Arthur Carvalho Walraven da Cunha
- Francisco Sérgio de Freitas Filho
- Jonas Costa Ferreira da Silva
- Jhonata Adam Silva Matias
- Pedro Paulo de Medeiros

1.2 French partners

Project COATI, I3S (CNRS & UNS) and INRIA

http://team.inria.fr/coati

COATI is a joint team between INRIA Sophia Antipolis - Méditerranée and the I3S laboratory (Informatique Signaux et Systèmes de Sophia Antipolis) which itself belongs to CNRS (Centre National de la Recherche Scientifique) and UNS (Université Nice Sophia Antipolis). Its research fields are Algorithmics, Discrete Mathematics, and Combinatorial Optimization, with applications mainly in telecommunication networks.

The main objectives of COATI are to design networks and communication algorithms. In order to meet these objectives, the team studies various fundamental theoretical problems in Discrete Mathematics, Graph Theory, Algorithmics, and Operations Research and develops applied techniques and tools, especially for Combinatorial Optimization and Computer Simulation. In particular, COATI used in the last years these theoretical and applied tools for specific problems such as energy efficiency in backhaul and backbone networks, routing reconfiguration in connection oriented networks (MPLS, WDM), traffic aggregation in SONET networks, compact routing in large-scale networks, evolution of the routing in case of any kind of topological modifications (maintenance operations, failures, capacity variations, etc.), survivability to single and multiple failures, etc. These specific problems often come from questions of their industrial partners (Alcatel-Lucent Bell-Labs, Orange labs, SME 3-Roam). Moreover, the combinatorial tools of COATI may be well applied to solve many other problems in various areas (transport, biology, resource allocation, chemistry, smart-grids, speleology, etc.) and the team has now collaborations in structural biology (with team ABS), transportation networks (with industrial partner Amadeus and Instant-System) and economics (with GREDEG laboratory).

The research done in COATI results in the production of advanced software such as Grph, and in the contribution to large open source software such as SageMath.

Participants

- **Frédéric Havet** (coordinator, Directeur de recherches, CNRS)
- Julien Bensmail (Maître de conférences, UNS)
- Frédéric Giroire (Chargé de recherches, CNRS)
- Nicolas Nisse (Chargé de recherches, INRIA)

G-SCOP Laboratory, Grenoble INP, CNRS and Université Grenoble Alpes

G-SCOP is a multidisciplinary laboratory which has been created to meet the scientific challenges imposed by the ongoing changes within the industrial world. The scope of the laboratory goes from the products conception to the production systems management and is based on strong skills in optimisation.

When we talk about design and production systems optimised management, the addition of powerful mathematical tools seems obvious. All of these techniques of modeling, optimisation and performance evaluation of new problems, make the upstream work essential in order to help create powerful tools in optimisation. G-SCOP has 6 fields of expertise, namely, Combinatorial Optimisation, Operational research and Production system, Management and Conduct of Production systems, Collaborative Design, Design Product Process and Information system design robust Products. This project involves the team of Combinatorial Optimisation of G-SCOP.

Participants

- **Frédéric Maffray** (Directeur de Recherche, CNRS)
- Nicolas Bousquet (Chargé de Recherche, CNRS)
- Alantha Newman (Chargé de Recherche, CNRS)
- Myriam Preissmann (Directeur de Recherche, CNRS)
- András Sebő (Directeur de Recherche, CNRS)
- Matěj Stehlík (MC, Université de Grenoble Alpes)
- Louis Esperet (Chargé de Recherche, CNRS)
- Benjamin Lévêque (Chargé de Recherche, CNRS)
- Zoltan Szigeti (Enseignant Chercheur, INP)

History of the collaboration between the teams

The collaboration between the teams has started almost 20 years ago when Ricardo Corrêa (one of the founders of ParGO) prepared his PhD thesis under the supervision of Afonso Ferreira, a former member of MASCOTTE (the former team composed by the members of COATI). Since then, the level of scientific exchanges between the two teams has just increased every year.

In the first years of the collaboration, several members of COATI regularly visited the Federal University of Ceará (in average three two-weeks visits per year) while the visits of the brazilian researchers have been less regular but of longer term. For instance, Ricardo Corrêa spent in Sophia-Antipolis 2 months in 2003 and 3 months in 2006 while Claudia Linhares Sales stayed in France for the whole academic year 2006/2007.

After this initial period, thanks to the support of several research projects that started to fund the partnership of these two groups, many mutual visits were possible. There were several short term visits of researchs of both teams and also many long term visits of students. Long term visits of french PhD students have been made to Fortaleza as well as long term visits of many brazilian students to MASCOTTE. We describe in detail in Section 1.2 all the visits of brazilian students to MASCOTTE/COATI.

We emphasize that these visits of students have been extremely fruitful for both groups and the corresponding universities that they belong to. There were five students who first spent few months as internship students of COATI during their master's course in UFC, and then they were back to France few months later to prepare their PhD thesis in France (three of these did their PhD thesis in a co-agreement between the two universities, under the co-supervision of a member of ParGO and a member of COATI). Another important fact to emphasize is that all these five students are now professors at the biggest universities in the state of Ceará and they are fully involved in graduation programs at their universities. More recently, other five brazilian students also have visited COATI for 3 months each.

Our partnership has been partly funded by an Equipe Associée INRIA (EWIN 2009-2011), by an INRIA-FUNCAP programme (ALERTE 2011-2013) and lastly by a FUNCAP-INRIA-CNRS programme (GAIATO 2014-2016) that is coordinated by Manoel Campêlo and Frédéric Havet and that ends soon.

After 20 years of exchanges, the groups are fully involved to each other. They share common research interests and they have a great interest to keep working on research together and overall to keep providing high level formation to several graduation students of both teams.

Another extremely important scientific contribution of this partnership is that we organized two workshops in the areas of interest of the teams: the *1st French-Brazilian Workshop on Graphs and Combinatorial Optimization*[1] held in Redonda, Ceará, Brazil, in May 2012 and more recently we organized the *2nd French-Brazilian Workshop on Graphs and Combinatorial Optimization* in the same place this year[2]. As part of this proposal, we plan to organize the third edition of this workshop in 2018.

On the other hand, the collaboration with G-SCOP started when Cláudia Linhares Sales did her PhD, finished in 1996, under the supervision of Claude Benzaken and Frédéric Maffray and their team (at the time they belonged to Artemis and LSDD whose union with other laboratories in Grenoble gave place to G-SCOP). Several years later, giving continuity to this collaboration, Ana S. Silva, another member of ParGO, also did her PhD under the supervision of Frédéric Maffray (in the Leibniz Laboratory). Along all these years, ParGO has received regular visits of Frédéric Maffray, András Sebö and Matej Stehlík. Actually, Matej Stehlík did a short pos-doctorate internship at ParGO in 2011 just before his got his position at University Grenoble Alpes.

Common students

In order to emphasize the collaboration between the groups in the formation of students, here we list those students that were supervised by the members of the different teams.

Students preceded by a $\star$ were co-supervised in their PhD thesis by a member of ParGO and a member of COATI; students preceded by a + obtained their master's degree at the Federal University of Ceará and they were awarded a brasilian PhD grant from CAPES to attend their PhD in COATI; students preceded by a $\triangle$ visited the COATI project as intership students while they were preparing their PhD's thesis under supervision of a member of ParGO; students preceded by a $\square$ visited the COATI project as intership students while they were doing their master's course under supervision of a member of ParGO; students preceded by a $\oplus$ were supervised in their MSc dissertation by a member of ParGO and did their PhD thesis under the supervision of member of G-SCOP.

- $\square$ Raul Wayne Teixeira Lopes. Currently preparing his PhD thesis under supervision of Victor Almeida Campos. Internship student in COATI from September to November 2016.

- $\square$ Sammuel Nascimento de Araújo. *Número de Dominação Romana em Grafos.* Master's dissertation under supervision of Rudini Sampaio. Internship student in COATI from May to July 2015.

- $\triangle$ Nicolas de Almeida Martins. *Problemas de Coloração de Grafos com Poucos P_4's.* Master's dissertation under joint supervision of Rudini Sampaio and Ana Shirley Silva. Now PhD student at Federal University of Ceará and professor at Universidade Estadual Vale do Acaraú. Internship student in COATI from May to July 2015.

- $\square$ Rennan Ferreira Dantas. *Problemas de Código de Identificação em Grides.* Master's dissertation under joint supervision of Rudini Sampaio and Frédéric Havet. Now PhD student and professor at Federal University of Ceará. Internship student in COATI from November 2013 to February 2014.

[1] `http://www-sop.inria.fr/mascotte/Events/GCO2012/`

[2] `www.lia.ufc.br/GCO2016`

- ☐ Claudio Soares de Carvalho Neto. *Embutimento de Árvores Orientadas em Digrafos*. Master's dissertation under joint supervision of Claudia Linhares Sales and Frédéric Havet. Now professor at Universidade Estadual Vale do Acaraú. Internship student in COATI from November 2013 to February 2014.

- +, ☐ Ana Karolinna Maia de Oliveira. *Subdivisions of Digraphs*. PhD thesis, University of Nice-Sophia Antipolis, 2014. Now professor at Federal University of Ceará. Internship student in COATI from March to June 2010.

- ⋆, ☐ Ronan Pardo Soares. *Pursuit-Evasion Games, Decompositions and Convexity on Graphs*. PhD thesis, University of Nice-Sophia Antipolis and Federal University of Ceará, 2013. Now professor at Federal University of Ceará. Internship student in COATI from February to May 2009.

- +, ☐ Leonardo Sampaio Rocha. *Algorithmic aspects of graph colouring heuristics*. PhD thesis, University of Nice-Sophia Antipolis, November 2012. Now professor at Universidade Estadual do Ceará. Internship student in COATI from January to March 2008.

- ⋆, ☐ Julio Cesar Silva Araujo. *Graph Coloring and Graph Convexity*. PhD thesis, University of Nice-Sophia Antipolis and Federal University of Ceará, September 2012. Now professor at Federal University of Ceará. Postdoc in COATI 2012-2013. Internship student in COATI from March to April 2009.

- ⋆ Napoleão Vieira Nepomuceno. *Network optimization for wireless microwave backhaul*. PhD thesis, Université de Nice-Sophia Antipolis and Federal University of Ceará, 2010. Now professor at University of Fortaleza.

- ⊕ Ana S. Silva. *b-colouring of tree-like graphs*. PhD thesis, Université de Grenoble, 2010. Now professor at Federal University of Ceará.

2 Scientific description of the collaboration

Let us first present some general headlines about the research we plan to develop, before describing the details of the problems we plan to study.

Our goal is to study the Computational Complexity of some network problems. We plan to design algorithms to solve these problems efficiently or to theoretically prove that such algorithms may not exist, under some theoretical hypothesis. In case such algorithms exist, we also plan to implement them and to test their performance under realistic instances.

Lots of these problems can be naturally modelled as graph-theoretical or combinatorial problems. Because realistic problems are very complex, we first study simplified models. These models have then to be more and more refined so as to be closer to reality. Our work can be decomposed into three tasks.

The first one is to obtain theoretical results about these problems. The goal of this task is to obtain algorithmically meaningful theoretical results such as: structural results, combinatorial bounds for optimization problems, and complexity results.

General Objective 1. *Obtain structural results, combinatorial bounds for optimization problems and computational complexity results.*

When we refer to structural results, we mean providing decomposition theorems or intrinsic properties of graphs in a given class that are often precious guides for designing efficient algorithms. Many of the problems we plan to study are optimization problems, i.e. problems where some parameter must be maximized or minimized. We plan to find combinatorial bounds for these problems that will enable us to apply cuts in branching algorithms or to establish approximation ratios for the value of the solutions that these algorithms provide. At last, we also plan to obtain complexity results by proving that some of these problems are NP-complete, or (non-)approximable under some factor, fixed parameter tractable, $W[i]$-hard, for some $i \in \mathbb{N}$, etc.

The second and most important task of this project is to design and analyse algorithms for these problems. Obviously, it depends on the previous task, since it is necessary to understand the complexity of the problem before providing algorithms for it, as well as the next task we mention that is to implement the algorithms and evaluate their behaviour for several sets of instances.

General Objective 2. *Design, analysis, implementation and evaluation of algorithms for networking problems.*

For both the first and the second tasks, we also plan to study the complexity of the considered problems under the viewpoint of Parameterized Complexity. This area is a relatively new domain in Theoretical Computer Science, which originated in the 90's when Downey and Fellows [99] laid the foundations for a fine-grained analysis of the complexity of computational problems. In a nutshell, the main idea is to go beyond the classical distinction between polynomial vs. NP-hard problems, and provide a more refined analysis of their complexity taking into account how they behave with respect to an additional information given in the input, usually called the *parameter*. This approach permits a better understanding of the hardness of the problems, as it allows to identify which aspects of the input, or the desired output, make a problem hard, and how this hardness can be overcome –or not– via the so-called *fixed-parameter tractable* algorithms (FPT algorithms for short).

Finally, the last task aims at testing the practical efficiency of the algorithms we design in order to compare them with existing algorithms. We are paying a particular attention to the reproductibility and to our experimentations, in particular by providing free access to our algorithms and data sets. ParGO and COATI have experienced developing common tools for implementation and experimentation. In particular, we developed a common interface to the two libraries Mascopt[3] and Parego. Nowadays, COATI is developing a new library of graph algorithms[4] as well as contributing to the SageMath framework.

General Objective 3. *Testing the efficiency of the proposed algorithms by comparing them to others in the literature.*

In the last years, we mainly focused on optimization and dynamic routing in wireless networks and on various channel assignment problems related to graph colouring. For the next few years, we plan to continue to work on some these problems, but also would like to work on some others. In the sequel, we detail the topics on which we will focus.

2.1 Graph colouring and allocation/scheduling problems

Wireless networks represent crucial infrastructures for modern information society. In a wireless radio network, a transmission is performed by emission of a radio signal from the antenna of a node, travels through a communication channel and is received by the antenna of another node. Interference between nearby transmissions prevents simultaneous communication between pairs of nodes that are sufficiently close to each other [90]. One way to cope with this conflict constraint is to use a time division multiple access (TDMA) communication protocol. In such a protocol, the communication in the network proceeds by the successive repetition of a sequence of transmission rounds, in each of which only noninterfering transmissions are allowed to take place. Such protocols can also be used in the particular case where each node not only communicates data relevant to itself, but also forwards packets sent by other nodes, thus enabling communication between distant parts of the network [90, 106]. Hence, nodes that cannot communicate directly can also exchange messages, since the routers relay the network packets sent by a node until they reach their destinations.

We study various problems of channel assignment/transmission scheduling that may be modelled as graph colouring problems. For each problem, the final aim is to design the most efficient algorithm possible. Most of these problems are NP-hard. So we may not expect to have polynomial-time algorithms to solve them. Hence we will try several approaches which may be seen as different compromises between the quality of the solution and the (theoretical) running time of the solutions: approximation algorithms, exact exponential-time algorithms, fixed-parameter tractable algorithms, heuristics, etc.

General Objective 4. *Find approximation, exact exponential-time, fixed-parameter tractable algorithms and heurists for variations of the graph colouring problems.*

Some years ago, we worked on a problem, posed by Alcatel Space Technologies, about sending information from a satellite to receivers on earth [40, 50, 58, 67]. Another promising technology that aims to reduce interference in wireless environments is cognitive radios. Those devices are able to detect unused licensed channels, also called White Spaces, and to reconfigure their operating characteristics in order to use these bands without causing harmful interference to licensed radios [85, 86]. The insertion, in a legacy network, of these cognitive radios using free licensed channels in a dynamic and opportunistic fashion characterizes a hybrid network. This hybrid network is composed by two types of link: (i) the legacy links, which have spectrum access guarantees in ISM unlicensed bands, and (ii) the cognitive radio links, which have unstable availability due to licensed radios activity. In addition, it achieves an improved spectrum

[3] `http://www-sop.inria.fr/mascotte/mascopt`
[4] `http://grph.inria.fr/`

utilization with respect to the legacy network. Because of their non-priority nature in spectrum access, cognitive radios are directly dependent on the activity of licensed devices. In a multi-hop network, depending on the white spaces available for cognitive radios, links may become temporarily unavailable. This problem leads to a dynamic network topology, where route discovery and maintenance is difficult. Such a dynamic aspect of the network yields constraints on the number of rounds in which some specific channels can be activated. Hence, considering these additional constraints, an interesting challenge is to:

Problem 1. *Determine the optimal set of wireless routers that should receive the additional cognitive radio interface in order to maximize the aggregate capacity or the end-to-end delay.*

Another problem models channel assignment in radio networks. A usual constraint is a minimum separation between frequencies which depends on the distance between the transmitters (vertices) to which they are assigned. A first model use the concept of $L(p_1, \ldots, p_k)$-labelling, which is positive integer-valued function f such that $|f(x) - f(y)| \geq p_i$ if the vertices x and y are at distance at most i in the graph. This model is the particular case (for powers of graphs) of a more general one in which the edge set of a graph is partitionned into k sets $E_1, \ldots, E_k$. The aim is then to find a $(p_1, \ldots, p_k)$-backbone colouring that is a function f such that $|f(x) - f(y)| \geq p_i$ if the edge xy is in E_i. Our goal is to:

Problem 2. *Find bounds and design algorithms for finding $L(p_1, \ldots, p_k)$-labelling or $(p_1, \ldots, p_k)$-backbone colouring of graphs.*

We are particularly interested in classes of graphs that arise in real-life, like planar graphs, disk graphs or line graphs. We gave a series of NP-hardness results as well as upper bounds for (p_1, p_2)-backbone colouring for planar graphs, when the set E_2 induces various types of forests (matching, galaxy, spanning tree, ...) [14, 28, 83, 84]. There are still many open problems and conjectures in the area.

In addition, time is unidirectional, past events cannot be reversed and the future is uncertain. So on-line algorithms must be designed. In addition, such algorithms form an elegant framework for analyzing algorithms with incomplete information or incomplete access to the input. Hence we will study on-line algorithms. We obtained many results [24, 36, 38, 49, 56, 63, 64] for classical proper colouring. We will now focus on colourings arising from the above-mentioned applications.

Finally, there is a problem that has recently been defined that involves two topics that will be studied in this project: colourings and directed graphs (this last one is described in Section 2.5). An orientation D of a non-oriented graph G is a function that assigns to each edge of G a direction. One may also see D is a digraph whose subjacent graph is G. An orientation D of G is *proper* if $d^-_D(v) \neq d^-_D(u)$, for every edge $uv \in G$. Given $k \in \mathbb{N}$, a *proper k-orientation* of G is a proper orientation D that satisfies $d^-_D(v) \leq k$, for every $v \in V(G)$. The *proper orientation number* of a graph G, denoted by $\overrightarrow{\chi}(G)$, is the minimum k such that G admits a proper k-orientation. This problem was defined in [88] and we already have two publications involving the members of both groups [7, 12]. There are still several questions about this problem to be studied. For instance, it is not known if there is a constant upper bound for planar graphs, there are no complexity results for more dense graphs like chordal graphs, there is no result concerning the parameterized complexity of determining whether $\overrightarrow{\chi}(G) \leq k$, when k is the parameter, etc.

Problem 3. *Determine the computational complexity of finding an optimal proper orientation of a chordal graph and deciding whether there exists a constant upper bound for the proper orientation number of a planar graph.*

Concerning the latter aspect mentioned above, we have recently made some project toward better understanding the parameterized complexity of this problem. Namely, we have considered a *weighted* version of the problem, namely in which the edges of G come equipped with positive weights, and the in-degree of a vertex is defined as the sum of the weights of the incoming arcs in the chosen orientation. The motivation for introducing weights is given by the work of Szeider [118], who considers a similar weighted orientation problem. Building on the techniques of Szeider [118], we have proved that the weighted version of determining the proper orientation number is W[1]-hard parameterized by the treewidth of the input graph. This opens up a number of questions concerning the complexity of this weighted version, and we plan to continue studying them.

2.2 Dectecting faults in multiprocessor networks and identifying codes in graphs

Identifying problems by tests is very common in various domains such as pattern recognition, group test in biology, failure detection, localization, or key searching in databases [97, 102, 107, 112] We are more specifically interested in a problem that models fault diagnosis in multiprocessor systems. In these systems, it may happen that some of the processors become faulty, in a way that depends on the purpose of the system. We wish to detect and replace such processors, so that the system can work properly. We assume that our hardware is of such a quality that, at any time, at most k of the processors of the system are faulty, where k is a fixed constant. Let us assume that each processor p of the system is able to run a procedure `test(p)`, which checks its own state as well as the state of its neighbouring processors $N(p)$. This procedure returns only binary information; e.g. 0 if p or a processor of its neighborhood $N(p)$ is faulty, and 1 otherwise. This information is returned to a central controller, which is not considered to be part of the system. Note that the procedure does not reveal the identity of the faulty processor: If $\texttt{test}(p)$ outputs 0, then all we can say is that p and/or some of its surrounding processors in $N(p)$ is faulty. We wish to devise a subset of processors C such that:

(1) If all the processors of C return 1, then none of the processors of the network is faulty.

(2) If at least one, but at most k of the processors are malfunctioning, then the central controller is able to locate them using C.

Formally, we model the multiprocessor system by a graph $G = (V, E)$, whose vertices are processors and whose edges are links between these processors. For a vertex $v \in V$, the *closed neighbourhood*, denoted $N[v]$, is $N(v) \cup \{v\}$. Let $C \subseteq V$ be a subset of vertices of G, and for all subsets of at most k vertices $X \subseteq V$, let us denote $I(X, C) = \bigcup_{x \in X} N[x] \cap C$. If all the $I(X, C)$'s are distinct, then we say that C *separates* the sets of at most k vertices of G, and that C is a *k-identifying code*.

Not every graph has a k-identifying code. In fact, a graph admits a k-identifying code if and only if for every pair of subsets $X \neq Y$, $|X|, |Y| \leq k$, we have $N[X] \neq N[Y]$, where $N[S]$ denotes $\bigcup_{s \in S} N[s]$. In the case, where G admits a k-identifying code, then $C = V$ is always a k-identifying code. We are usually interested in finding a k-identifying code of minimum cardinality, or when the graph is infinite, of minimum density.

In a recent work [76], we used the discharging method (which we intensively used and develop in our work on graph colouring) to obtain optimal 1-identifying code in the square grid of height 3. The use of the discharging method for finding and proving (near) optimality of identifying codes seems very promising. We also used it [10] to obtain (near) optimal 1-identifying codes in the triangular grids of bounded height. We now plan to focus to k-identifying of those grids as well as other grids.

We also would like to investigate directed variants of the problems. The first one is the very analogue of the undirected problems for directed graphs, where the closed neighborhood is replaced by the *closed out-neighbourhood*. The second one consists in finding an orientation of a given undirected graph which has an identifying code of smallest possible size.

Problem 4. *Determine bounds and algorithms to optimal identifying codes in directed graphs.*

Hopefully, we would be able to find a general method, possibly computer-assisted, to find optimal identifying codes, for large classes of (directed) graphs.

2.3 Graph searching

Pursuit-evasion games involve a team of mobile agents that aims at capturing a set of escaping agents that hide in a network modelled by a graph. These are natural games arising from a wide range of applications, from the practical ones such as search and rescue (e.g., rescuing a speleologist in a cave, [93, 94], surveillance, monitoring, military strategy to trajectory tracking, etc. to the handling of abstract mathematical and theoretical computer science concepts (e.g., Graph Minor Theory by Robertson and Seymour [115]). Hence, Pursuit-evasion games have been studied by various divergent disciplines, e.g., graph theory, differential games, robotics, control theory, geometric algorithms, etc.

There are many variants of Pursuit-evasion games studied in the literature. During his Ph.D. thesis, Ronan P. Soares studied the so-called graph searching games for their application to routing in communication networks [11, 30, 32].

We aim at studying the cops and robber games where a set of cops (controlled by one player) tries to capture the robber (controlled by the opposing player) [92]. The cops and the robber are restricted to the vertices of a graph and they move each round to neighbouring vertices. The smallest number of cops needed to capture the robber is the *cop number* $c(G)$ of the graph G. Such a simple-sounding game leads to a complex theory with many results and beautiful conjectures.

An interesting result, due to Aigner and Fromme [89], is the fact that, in every planar graph, the cop number is at most 3. Moreover, Quilliot [113] proved that, in every graph with genus g, the cop number is at most $2g + 3$ (the genus of a planar graph is 0) and Shroeder improved this to $3g/2 + 3$ [117]. The main conjecture in this topic is that the cop number is at most $c(G) \leq g + 3$ [117].

Problem 5. *Improve upper bounds to the cop-number of a graph of bounded genus.*

Another interesting conjecture, due to Meyniel [98], states that, if G has n vertices, then the cop number is $c(G) = O(\sqrt{n})$. That is, for n sufficiently large, there is a constant $d > 0$ such that $c(G) \leq d\sqrt{n}$. The best upper bound known so far is that $c(G) = O(\frac{n}{2^{\sqrt{\log n}}})$ in any n-node graph G [116]. This illustrates how far we are from proving the Meyniel's conjecture, even the so-called *soft Meyniel's conjecture*, which states that, for a fixed constant $d > 0$, $c(n) = O(n^{1-d})$.

Problem 6. *Determine whether there is a constant $d > 0$ such that $c(G) \leq d\sqrt{n}$, for n sufficiently large.*

Finally the question of approximating the cop-number of a graph is interesting. It is known that the cop-number problem is NP-hard and that no polynomial-time algorithm with approximation ratio $O(\log n)$ exists unless $P = NP$ [82]. The problem is even known to be EXPTIME-complete [105]. However, it is not known whether the cop-number can be approximated up to a ratio $O(n^{1-\varepsilon})$ for some $\varepsilon > 0$.

Problem 7. *Determine whether the cop-number can be approximated up to a ratio $O(n^{1-\varepsilon})$ for some $\varepsilon > 0$.*

The PhD student Nicolas Martins, from Universidade Federal do Ceará, is studying these problems and has already obtained some results together with COATI members [9]. Nicolas Nisse (COATI) is also involved in the study of these games. From October 2016, he is supervising the Ph.D. thesis of Fionn Mc Inerney on this topic.

2.4 Graph convexity

Another area that we plan to work during this project concerns also some dynamic process in the graph, as in Section 2.3. In the sequel, we described it formally, but before let us present some intuition about what practical applications could be modelled by these problems.

One may see such problems in Graph Convexity as infection problems in a graph. Given a set of initially infected vertices, one may define an infection function that says that a vertex that as two adjacent infected neighbours will become infected, or another infection function could be the one in which every vertex that belongs to a shortest path between two infected neighbours will become infected. Given such infection process, one may wonder what is the minimum size of a set that we shall infect at the beggining of the process to ensure that the whole graph is going to be fully infected at the end. Such problem can be modelled as determining a particular parameter of a graph convexity. Several practical applications that could be modelled by such problems as disease spreading, rumor spreading in social networks, etc.

A classical example of convexity is the one defined in Euclidean spaces. In an Euclidean space E, a set $S \subseteq E$ is said *convex* if for any two points x and y of S, $[x, y] \subseteq S$, i.e., the set of points lying in the straight line segment between x and y also belongs to S. Note that if two convex sets $X, Y \subseteq E$ contain a given set $S \subseteq E$ of points, then their intersection $X \cap Y$ is also a convex set of E containing S. Hence, we can define the *convex hull* of S as the minimum convex set that contains S. Reciprocally, given a convex set S of E, a *hull set* of S is any subset S' of S such that S is the convex hull of S'. A naive way to compute the convex hull H of a set S consists in starting with $H = S$ and, while it is possible, adding $[x, y]$ to H for any $x, y \in H$. However there exist more efficient algorithms. For instance, for any set S of a d-dimensional euclidean space, the *gift wrapping algorithm* computes the convex hull and a minimum hull set of S in polynomial-time in the size of S (d being fixed). For more results concerning the convexity in Euclidean spaces, we refer to [114].

In order to capture the abstract notion of convexity, [100] defines an *alignment* over a set X as a family $\mathcal{C}$ of subsets of X that is closed under intersection and that contains both X and the empty set. The members of $\mathcal{C}$ are called the *convex sets* of X. The pair $(X, \mathcal{C})$ is then called an *aligned space*. An example of aligned space $(E, \mathcal{C})$ is the one where E is an Euclidean space and $\mathcal{C} = \{H \subseteq E \ : \ \forall x, y \in H,\ [x, y] \subseteq H\}$. Given an aligned space $(X, \mathcal{C})$, the definitions of convex hull and hull set are generalized as follows. For any $S \subseteq X$, the *convex hull* of S is the smallest member of $\mathcal{C}$ containing S. For any $S \in \mathcal{C}$, a *hull set* of S is a set $S' \subseteq S$ such that S is the convex hull of S'.

Various notions of convexity can be defined in graphs as specific alignments over the set of vertices. For instance, let us present some notions related to the *geodetic convexity* of graphs. Let $G = (V, E)$ be a connected undirected graph. For any $u, v \in V$, let the *closed interval* $I[u, v]$ of u and v be the the set of vertices that belong to some shortest (u, v)-path. The closed interval of a set of vertices can be seen as an analog to segments in Euclidian spaces. For any $S \subseteq V$, let $I[S] = \bigcup_{u,v \in S} I[u, v]$. A subset $S \subseteq V$ is *geodesically convex* if $I[S] = S$. Here convexity refers to the geodesic variant, although there are several others defined in the literature concerning induced paths, paths on three vertices, etc. In other words, a subset S is convex if, for any

$u, v \in S$ and for any shortest (u, v)-path P, $V(P) \subseteq S$. That is, the geodetic convexity can be defined as the alignment $\mathcal{C}$ over V where $\mathcal{C} = \{S \subseteq V \ : \ I[S] = S\}$. There are several parameters to be studied in this domain. The *hull number* of a graph G is the minimum cardinality of a set $S \subseteq V(G)$ such that $H[S] = V(G)$. The *interval number* is simply the minimum cardinality of a set $S \subseteq V(G)$ satisfying $I[S] = V(G)$.

An interesting research direction in graph convexity concerns the parameterized complexity of the corresponding problems. More precisely, in the PhD thesis of Thiago Marcillon, advised by Rudini Sampaio from the ParGO team, several results about the parameterized complexity of different variants of convexity problems have been provided [110, 111]. Nevertheless, a few important open questions remain open, and we plan to work on them in this project. For instance, it is not known whether several convexity problems are FPT of W[1]-hard when taking as the parameter the pathwidth of the input graph, which is a notorious graph parameter more restrictive than treewidth.

Problem 8. *What is the parameterized complexity of determining the geodetic hull number of a graph G, when the parameter is the pathwidth of G?*

There are two other promising research avenues that we will follow in this project. The first one is a new graph convexity that we call Cycle Convexity whose motivation comes from Knot Theory. There are some preliminary results by authors of ParGO and COATI on the interval number in this convexity that will be submitted soon.

The second one is to study graph convexity parameters for directed graphs. Although several works study the undirected case, we found only five papers in the literature concerning graph convexity parameters in directed graphs.

General Objective 5. *Study graph convexity parameters for different graph convexities for directed graphs.*

2.5 Directed graphs

Significant advances in algorithms on graphs (as in various other combinatorial settings) seem to be closely allied with deep combinatorial structure theory. Hence one of our goal is to obtain new results in graph theory, which will hopefully be of algorithmic use.

Depending on the application, the modelling gives a problem on undirected graphs or directed graphs (digraphs). However, for various reasons, undirected graphs have been studied much more extensively than directed graphs. It seems that the main reason is that undirected graphs form, in a sense, a special class of directed graphs (symmetric digraphs) and hence problems that can be formulated for both directed and undirected graphs are often easier for the latter. While the theory of undirected graphs can be considered now as a developed field of mathematics consisting of several deep areas of its own, the theory of digraphs is still very much an emerging area of research. Our goal is to develop digraph theory and to make progress on some long-standing conjectures and problems.

In particular, we would like to focus on subdivision of digraphs, from both a structural and algorithmic point on view.

Firstly, we are interested in which parameters would guarantee the existence of a subdivision of a fixed digraph F.

General Objective 6. *Determine graph parameters that ensure a given graph to have a subdivision of a fixed digraph F.*

For undirected graphs, Mader (1967) established that every graph with sufficiently large minimum degree contains a subdivision of K_k, the complete graph on k vertices, and this implies that every graph with sufficiently large chromatic number chromatic number or connectivity contains a subdivision of K_k. We are interested in directed analogues of those statements: Let γ be a digraph parameter and let F be a digraph. F is **γ-maderian** if there exists c such that every digraph D with $\gamma(D) \geq c$ contains a subdivision of F. The smallest c for which it is true is called the *γ-value* of F. We are interested in γ-maderian digraphs where γ is either the minimum out-degree δ^+, the minimum in-degree δ^-, the minimum semi-degree $\delta^0 = \min\{\delta^+, \delta^-\}$, the chromatic number χ, the dichromatic number $\vec{\chi}$. An important conjecture in this field is due to Mader (1985) and states that every acyclic digraph is δ^+-maderian.

Problem 9 (Mader, 1985)**.** *Is every acyclic digraph δ^+-maderian?*

We would like to tackle this conjecture. More generally, we would like to characterize the δ^+-maderian digraphs as there are non-acyclic digraphs (directed cycles for example) which are δ^+-maderian.

We recently proved a result [78] implying that the χ-maderian digraphs are the oriented forests. However determining their χ-value is still open. Burr conjectured in 1980 that every digraph with chromatic number $2k-2$ contains every oriented tree of order k as a subdigraph. In [25], we gave some new results towards this conjecture. We are still pursuing our work on this conjecture. An interesting first step towards both Burr's conjecture and determining the χ-value of oriented forest would be to prove the following conjecture: *Every digraph with chromatic number $2k-2$ contains a subdivision of every oriented tree of order k.*

On the algorithmic point of view, an important problem is the one, called F-Subdivision, of deciding if a digraph D contains a subdivision (not necessarily induced) of a fixed digraph F. Contrary to the undirected case, where the problem is always polynomial-time solvable by Robertson and Seymour Graph Minors Theory, we found in [15] many digraphs F for which the problem is NP-complete. On the other hand, we also gave polynomial-time algorithms to solve the problem for many others digraphs. This leads us to conjecturing that there is a sharp dichotomy between NP-complete and polynomial-time solvable instances: according to this conjecture, there are only two kinds of digraphs F: the *hard* ones, for which F-Subdivision is NP-complete, and the *tractable* ones, for which it is polynomial-time solvable. However, there is no very clear picture of which graphs should be tractable and which ones should be hard, although some conjectures give some outline. Motivated by directed treewidth and a conjecture of Johnson et al. [103], Seymour posed the conjecture that F-Subdivision is polynomial-time solvable when F is a planar digraph with no big vertices. We [15] proposed the following sort of counterpart: F-Subdivision *is NP-complete for every non-planar digraph.* We would like to attack the preceding three conjectures and better understand what makes a digraph F tractable or hard. A first step would be to determine the complexity of F-Subdivision for several classes of digraphs. Many graph classes are of interest like oriented trees, oriented cycles. To attack the conjecture of Bang-Jensen et al. [15], looking at F-Subdivision when F is any orientation of $K_{3,3}$ is a natural question. We emphasize that part of these results were obtained during the PhD thesis of Ana Karolinna Maia, under the supervision of Frédéric Havet.

Recently, we have started exploring the parameterized complexity of the F-Subdivision problem in the particular relevant case where F is a spindle [1], taking $|V(F)|$ as the parameter. This was posed as an open problem by Bang-Jensen et al. [15], who proved that the problem can be solved on n-vertex graphs in time $n^{O(|V(F)|)}$. More precisely, for two positive integers k and ℓ, a $(k \times \ell)$*-spindle* is the union of k pairwise internally vertex-disjoint directed (u, v)-paths with ℓ edges between two vertices u and v. Several variants of the F-Subdivision problem when F is a spindle generalize the classical Maximum Flow and Longest Path problems.

We have achieved a sharp dichotomy of the complexity of F-Subdivision with respect to the length in the paths of the spindle, when the objective is to maximize the number of paths. On the other hand, for the particular case $k = 2$, we have proved that the problem is FPT when parameterized by the total length of both paths. It remains open to decide whether the F-Subdivision problem for a general spindle F is FPT of W[1]-hard, and we plan to continue working on this problem.

Problem 10. *Decide whether the F-Subdivision problem for a general spindle F is FPT or W[1]-hard.*

3 References

Common publications (since 1997)

[1] J. Araújo, V. A. Campos, A. K. Maia, I. Sau, and A. Silva. On the complexity of finding internally vertex-disjoint long directed paths. *Submitted*, 2017.

[2] F. Havet, A. K. Maia, B. Mohar. Finding a subdivision of a prescribed digraph of order 4, *Submitted* [Research Report] RR-8773, INRIA – I3S. 2015.

2017

[3] J. Araújo, J. Baste, and I. Sau. Ruling out FPT algorithms for Weighted Coloring on forests. *Proceedings of LAGOS'17 – IX Latin-American Algorithms, Graphs and Optimization Symposium*, Electronic Notes in Discrete Mathematics, to appear, Sep. 2017, Marseille, France.

[4] N. Cohen, F. Havet, W. Lochet and R. Lopes. Bispindles in strongly connected digraphs with large chromatic number. *Proceedings of LAGOS'17 – IX Latin-American Algorithms, Graphs and Optimization Symposium*, Electronic Notes in Discrete Mathematics, to appear, Sep. 2017, Marseille, France.

[5] R. Dantas, F. Havet, R. Sampaio. Identifying codes for infinite triangular grids with a finite number of rows. *Discrete Mathematics*, 340(7): 1584–1597, 2017.

[6] R. Dantas, F. Havet, R. Sampaio. Minimum density of identifying codes of king grids. *Proceedings of LAGOS'17 – IX Latin-American Algorithms, Graphs and Optimization Symposium*, Electronic Notes in Discrete Mathematics, to appear, Sep. 2017, Marseille, France.

2016

[7] J. Araujo, F. Havet, C. Linhares Sales, A. Silva. Proper orientation of cacti. *Theoretical Computer Science*, 639:14-25, 2016.

[8] J. Araujo, G. Morel, L. Sampaio, R. Soares, V. Weber, *Hull number: P_5-free graphs and reduction rules*, Discrete Applied Mathematics, Volume 210, 10 September 2016, Pages 171-175.[5]

[9] N. Cohen, M. Hilaire, N. A. Martins, N. Nisse and S. Pérennes. Spy-Game on Graphs. *Proceedings of 8th International Conference on Fun with Algorithms, FUN 2016*, 10:1–10:16, 2016.

[10] R. Dantas, F. Havet, R. Sampaio. Identifying codes for infinite triangular grids with a finite number of rows. *Proceedings of BGW'16 – Bordeaux Graph Workshop* Nov 2016, Bordeaux, France.

[11] N. Nisse, R. Soares. On the Monotonicity of Process Number. *Discrete Applied Mathematics*. 210:103-111, 2016.

[5] G. Morel and V. Weber were members of COATI when this paper was submitted.

2015

[12] J. Araujo, N. Cohen, S. F. De Rezende, F. Havet, P. Moura. On the proper orientation number of bipartite graphs. *Theoretical Computer Science*, 566: 59-75, 2015.

[13] J. Araujo, F. Giroire, J. Moulierac, Y. Liu, R. Modrzejewski. Energy Efficient Content Distribution. *The Computer Journal*, 2015, pp.16.

[14] J. Araujo, F. Havet, Mathieu Schmitt. Steinberg-like theorems for backbone colouring. *Proceedings of LAGOS'15 – VIII Latin-American Algorithms, Graphs and Optimization Symposium*, Electronic Notes in Discrete Mathematics 39:223-229, Mar.2015, Beberibe, Brasil.

[15] J. Bang-Jensen, F. Havet, A. K. Maia. Finding a subdivision of a digraph. *Theoretical Computer Science*, 562, 2015.

[16] F. Giroire, I. Lamprou, D. Mazauric, N. Nisse, S. Pérennes, R. Soares. Connected Surveillance Game. *Theoretical Computer Science* 584: 131-143, 2015.

[17] A. Kodjo, B. Jaumard, N. Nepomuceno, M. Kaddour, D. Coudert. Dimensioning microwave wireless networks. *Proceedings of ICC 2015 – IEEE International Conference on Communications*, Jun 2015, London, United Kingdom. IEEE, pp.2803 – 2809, 2015.

2014

[18] J. Araujo, J-C. Bermond, G. Ducoffe. Eulerian and Hamiltonian dicycles in directed hypergraphs. *Discrete Mathematics, Algorithms and Applications*, 06, 2014.

[19] J. Araujo, N. Cohen, S. De Rezende, F. Havet, P. Moura. On the proper orientation number of bipartite graphs. *Proceedings of ICGT2014 – 9th International colloquium on graph theory and combinatorics*, Jun 2014, Grenoble, France.

[20] J. Araujo, N. Nisse and S. Pérennes. Weighted Coloring in Trees. *SIAM J. Discrete Math.* 28(4): 2029-2041, 2014.

[21] J. Araujo, N. Nisse, S. Pérennes. Weighted Coloring in Trees. *Proceedings of STACS 2014 – 31st Symposium on Theoretical Aspects of Computer Science*, Mar 2014, Lyon, France. Dagstuhl Publishing, pp.75-86, 2014.

[22] V. Campos, C. Linhares Sales, A. K. Maia, R. Sampaio. Maximization Coloring Problems on graphs with few P4s. Discrete Applied Mathematics 164 (2):539-546, 2014.[6]

[23] G. Classen, D. Coudert, A. Koster, N. Nepomuceno. Chance-Constrained Optimization of Reliable Fixed Broadband Wireless Networks. *INFORMS Journal on Computing* 26 (4):893-909, 2014.

[24] F. Havet, A K. Maia, M. Yu. Complexity of greedy edge-colouring, *Journal of the Brazilian Computer Society* 21 (18), 2015.

2013

[6] A.K. Maia was a member of COATI when this paper was published.

[25] L. Addario-Berry, F. Havet, C. Linhares Sales, B. Reed, and S. Thomassé. Oriented trees in digraphs. *Discrete Mathematics*, 313(8):967-974, 2013.

[26] J. Araujo, V. Campos, F. Giroire, N. Nisse, L. Sampaio and R. Soares. On the hull number of some graph classes. *Theoretical Computer Science*, 475:1-12, 2013.

[27] J. Araujo, G. Morel, L. Sampaio, S. Soares, and V. Weber. Hull number: P_5-free graphs and reduction rules. In *VII Latin-American Algorithms, Graphs and Optimization Symposium (LAGOS)*, Playa del Carmen, Mexico, April 2013.[7]

[28] V. Campos, F. Havet, R. Sampaio, and A.-S. F. Silva. Backbone colouring: tree backbones with small diameter in planar graphs. *Theoretical Computer Science*, 487(0):50-64, 2013.

[29] V. Campos, A. Gyárfás, F. Havet, C. Linhares Sales, F. Maffray. *New bounds on the Grundy number of products of graphs.* Journal of Graph Theory 71 (2012) 78-88.

[30] F. Giroire, D. Mazauric, N. Nisse, S. Pérennes, and R. P. Soares. Connected surveillance game. *20th International Colloquium on Structural Information and Communication Complexity (SIROCCO)*, Lecture Notes in Computer Science. Springer, 2013.

[31] F. Maffray, A. Silva. b-colouring the Cartesian product of trees and some other graphs. *Discrete Applied Mathematics* 161 (2013) 650-669.

[32] N. Nisse and R. Soares. On The Monotonicity of Process Number. In *Proceedings of the Latin-American Algorithms, Graphs and Optimization Symposium (LAGOS'13), Electronic Notes in Discrete Mathematics*, Playa del Carmen, Mexico, April 2013.

2012

[33] J. Araujo, J-C. Bermond, F. Giroire, F. Havet, D. Mazauric, and R. Modrzejewski. Weighted improper colouring. *Journal of Discrete Algorithms*, 16:53-66, 2012.

[34] J. Araujo, N. Cohen, F. Giroire, and F. Havet. Good edge-labelling of graphs. *Discrete Applied Mathematics*, 160(18):2501-2513, 2012.

[35] J. Araujo and C. Linhares Sales. On the Grundy number of graphs with few P4's. Discrete Applied Mathematics, 160(18):2514-2522, 2012.[8]

[36] V. Campos, A. Gyárfás, F. Havet, C. Linhares-Sales, and F. Maffray. New bounds on the Grundy number of products of graphs. *Journal of Graph Theory*, 71(1):78-88, 2012.

[37] S. Dantas, F. Maffray, A. Silva. 2k2-Partition of some classes of graphs. *Discrete Applied Mathematics* 160 (2012) 2662-2668.

[38] F. Havet, C. Linhares Sales, and L. Sampaio. b-coloring of tight graphs. *Discrete Applied Mathematics*, 160(18):2709-2715, 2012.

[39] F. Maffray, A. Silva. b-colouring outerplanar graphs with large girth. *Discrete Mathematics* 312 (2012) 1796-1803.

2011

[7] G. Morel and V. Weber were members of COATI when this paper was presented.

[8] J. Araujo was a member of MASCOTTE (former COATI) when this work was published.

[40] J. Araujo, J-C. Bermond, F. Giroire, F. Havet, D. Mazauric, and R. Modrzejewski. Weighted Improper Colouring. In *Proceedings of International Workshop on Combinatorial Algorithms (IWOCA'11)*, Victoria, Canada, June 2011. Springer-Verlag.

[41] J. Araujo, V. Campos, F. Giroire, L. Sampaio, and R. Soares. On the hull number of some graph classes. In *Proceedings of European Conference on Combinatorics, Graph Theory and Applications (EuroComb'11)*, Budapest, Hungary, September 2011.

[42] V. Campos and F. Havet. 5-choosability of graphs with 2 crossings. INRIA research report RR-7618, 2011.

[43] G. Classen, D. Coudert, A. Koster, and N. Nepomuceno. Bandwidth assignment for reliable fixed broadband wireless networks. In *12th IEEE International Symposium on a World of Wireless Mobile and Multimedia Networks (WoWMoM)*, Lucca, Italy, pages 1–6, June 2011.

[44] G. Claßen, D. Coudert, A. M. C. A. Koster, and N. Nepomuceno. A chance-constrained model & cutting planes for fixed broadband wireless networks. In *5th International Network Optimization Conference (INOC 2011)*, volume 6701 of *LNCS*, pages 37–42. Springer, 2011.

[45] N. Cohen, D. Coudert, D. Mazauric, N. Nepomuceno, and N. Nisse. Tradeoffs in process strategy games with application in the WDM reconfiguration problem. *Theoretical Computer Science*, 412(35):4675–4687, 2011.

[46] D. Coudert, N. Nepomuceno, and I. Tahiri. Energy saving in fixed wireless broadband networks. In *International Network Optimization Conference (INOC), Lecture Notes in Computer Science*, Hamburg, Germany, June 2011.

[47] D. Coudert, N. Nepomuceno, and I. Tahiri. Optimisation de la consommation énergétique dans les réseaux sans fil fixes. In *Pascal Ducourthial, Bertrand et Felber, editor, 13es Rencontres Francophones sur les Aspects Algorithmiques de Télécommunications (AlgoTel)*, Cap Estérel France, May 2011.

2010

[48] J. Araujo, C. Linhares Sales, and I. Sau. Weighted coloring on P_4-sparse graphs. In *Proceedings of JDIR 2010)*, Sophia Antipolis, France, pages 33–38, March 2010.[9]

[49] M. Asté, F. Havet and C. Linhares-Sales. Grundy number and products of graphs. *Discrete Mathematics*, 310(9):1482–1490, 2010.

[50] J.-C. Bermond, F. Havet, F. Huc, and C. Linhares-Sales. Improper colouring of weighted grid and hexagonal graphs. *Discrete Mathematics, Algorithms and Applications*, 2(3):395–411, 2010.

[51] N. Cohen, D. Coudert, D. Mazauric, N. Nepomuceno, and N. Nisse. Tradeoffs in process strategy games with application in the WDM reconfiguration problem. In *Proceedings of FUN 2010*, LNCS 6099, Ischia Island, Italy, pages 121-132, June 2010.

[52] N. Cohen, D. Coudert, D. Mazauric, N. Nepomuceno, and N. Nisse. Tradeoffs in routing reconfiguration problems. In *Proceedings of Algotel 2010*, Belle Dune France.

[9] J. Araujo was a member of MASCOTTE (former COATI) when this work was published while I. Sau was a PhD student of MASCOTTE when this work was prepared.

[53] D. Coudert, N. Nepomuceno, and H. Rivano. Power-Efficient Radio Configuration in Fixed Broadband Wireless Networks. *Computer Communications, Special Section on Hot Topics in Mesh Networking*, 33(8):898–906, 2010.

[54] F. Havet and L. Sampaio. On the Grundy number of a graph. In Proceedings of the International Symposium on Parameterized and Exact Computation(IPEC), number 6478 of Lecture Notes on Computer science, pages 170-179, December 2010.

2009

[55] J. Araujo and N. Cohen and F. Giroire and F. Havet. Good edge-labelling of graphs. In *V Latin-American Algorithms, Graphs and Optimization Symposium (LAGOS 2009), Electronic Notes on Discrete Mathematics*, 2009.

[56] J. Araujo and C. Linhares Sales Grundy number on P4-classes. In *V Latin-American Algorithms, Graphs and Optimization Symposium (LAGOS 2009), Electronic Notes on Discrete Mathematics*, 2009.

[57] J.-C. Bermond, R. Corrêa and J. Yu. Optimal gathering algorithms on paths under interference constraints. *Discrete Mathematics*, 309(18):5574-5587, 2009.

[58] R. Corrêa, F. Havet and J.-S. Sereni, About a Brooks-type theorem for improper colouring. *Australasian Journal of Combinatorics*, 43:219–230, 2009.

[59] D. Coudert, N. Nepomuceno, and H. Rivano. Minimizing energy consumption by power-efficient radio configuration in fixed broadband wireless networks. In *Proceedings of IEEE HotMESH*, Kos, Greece, June 2009.

[60] D. Coudert, N. Nepomuceno, and H. Rivano. Joint optimization of routing and radio configuration in fixed wireless networks. In *Proceedings of AlgoTel'09*, Carry le Rouet, June 2009.

[61] F. Havet and C. Linhares-Sales. Combinatória e Problemas em Redes de Telecomunicaçoes. In *Proceedings of Colibiri 2009*, Rio Grande do Sul, Brasil, July 2009.

[62] C.T. Hoang, C. Linhares Sales, F. Maffray. On minimally b-imperfect graphs. *Discrete Applied Mathematics* 157 (2009) 3519-3530.

[63] C. Linhares Sales and L. Sampaio. b-coloring of m-tight graphs. In *V Latin-American Algorithms, Graphs and Optimization Symposium (LAGOS 2009), Electronic Notes on Discrete Mathematics*, 2009.[10]

2008

[64] M. Asté, F. Havet and C. Linhares-Sales. Grundy number and lexicographic product of graphs. In *Proceedings of ROGICS'08 (May 2008)*, Mahdia, Tunisia, 2008.

[65] D. Coudert, N. Nepomuceno, and H. Rivano. Power-efficient radio configuration in fixed broadband wireless networks. *Computer Communications, (Special Section on Hot Topics in Mesh Networking)* 33(8):898-906, 2010.

[10] L. Sampaio was a member of MASCOTTE (former COATI) when this work was presented.

[66] C. Linhares Sales, F. Maffray, B. Reed. On planar quasi-parity graphs. SIAM Journal on Discrete Mathematics 22 (2008) 329-347.

2007

[67] J.-C. Bermond, F. Havet, F. Huc, and C. Linhares-Sales. Allocation de fréquences et coloration impropre des graphes hexagonaux pondérés. In *Proceedings of ALGOTEL'2007)*, Ile d'Oléron, France, Mai 2007.

[68] F. Huc, C. Linhares-Sales and H. Rivano. The Proportional Colouring Problem: Optimizing Buffers in Wireless Mesh Networks In *Proceedings of LAGOS 07*, Puerto Varas, Chile, November 2007.

2004

[69] C. Linhares Sales, F. Maffray. On dart-free perfectly contractile graphs. *Theoretical Computer Science* 321 (2004), 171-194.

2003

[70] C. Linhares Sales, F. Maffray. Even pairs in square-free Berge graphs. *Matemática Contemporânea* 25 (2003), 161-176.

2001

[71] C. Linhares Sales, F. Maffray, B.A. Reed. Recognizing planar strict quasi-parity graphs. *Graphs and Combinatorics* 17 (2001), 745-757.

[72] H. Everett, C.M.H. de Figueiredo, C. Linhares Sales, F. Maffray, O. Porto, B.A. Reed. Even pairs. In: *Perfect graphs*, J.L. Ramirez-Alfonsin and B.A. Reed Eds., J. Wiley (2001), 67-92.

1998

[73] C. Linhares-Sales, F. Maffray. Even pairs in claw-free perfect graphs. *Journal of Combinatorial Theory B*, 74 (1998), 169-191.

1997

[74] H. Everett, C.M.H. de Figueiredo, C. Linhares-Sales, F. Maffray, O. Porto, B. Reed. Path parity and perfection. *Discrete Mathematics*, 165-166 (1997), 233-252.

[75] C. Linhares-Sales, F. Maffray, B. Reed. On planar perfectly contractile graphs. *Graphs and Combinatorics*, 13 (1997), 167-187.

Other relevant publications of the partners

[76] M. Bouznif, F.Havet, and M. Preissman. Minimum-density identifying codes in square grids. *In preparation.*

[77] M. B. Campêlo Neto, V. A. Campos, R. C. Corrêa, D. Delle Donne, J. Marenco, and M. Mydlarz. A polyhedral study of the maximum stable set problem with weights on vertex-subsets. *Discrete Applied Mathematics*, **210** (2015), 223–234.

[78] N. Cohen, F. Havet, W. Lochet, and N. Nisse. Subdivisions of oriented cycles in digraphs with large chromatic number *Technical report* RR-8865, INRIA, 2016.

[79] R. C. Corrêa, D. Delle Donne, I. Koch, and J. Marenco. General cut-generating procedures for the stable set polytope. *Discrete Applied Mathematics*, to appear, 2017.

[80] R. C. Corrêa, P. M. S. Farias, and C. P. de Souza. Insertion and sorting in a sequence of numbers minimizing the maximum sum of a contiguous subsequence. *J. Discrete Algorithms*, 21:1-10, 2013.

[81] R. C. Corrêa and P. M. Farias. Linear time computation of the maximal linear and circular sums of multiple independent insertions into a sequence. *Theoretical Computer Science*, **661** (2017), 8–17.

[82] F. Fomin, P. Golovach, J. Kratochvíl, and N. Nisse. Pursuing fast robber in graphs, *Theoretical Computer Science* **411** (2010), 1167–1181.

[83] F. Havet and A. D. King. List circular backbone colouring. *Discrete Mathematics and Theoretical Computer Science*, vol. 16:1, 2014, 89–104.

[84] F. Havet, A. D. King, M. Liedloff, and I. Todinca. (Circular) backbone colouring: tree backbones in planar graphs. *Technical report* RR-8152, INRIA, November 2012.

[85] M. W. R. Silva, *Redes cognitivas com oportunidades dinâmicas de acesso ao espectro*. PhD Thesis, COPPE/UFRJ - Brazil, 2011.

[86] M. W. R. Silva, J. F. Rezende. Routing in Hybrid Mesh Networks of Cognitive and IEEE 802.11 Radios. *IEEE Global Communications Conference*, 2010.

Other references

[87] 3Roam. 3Roam website, 2012.

[88] A. Ahadi, A. Dehghan, The complexity of the proper orientation number, Information Processing Letters 113 (19-21) (2013) 799–803.

[89] M. Aigner and M. Fromme. A game of cops and robbers, *Discrete Applied Mathematics* **8** (1984), 1–11.

[90] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran et al. NeXt Generation/Dynamic Spectrum Access/Cognitive Radio Wireless Networks: A Survey, *Computer Networks*, **13(50)** (2006).

[91] H. Anderson. *Fixed Broadband Wireless System Design.* John Wiley & Sons, 2003.

[92] A. Bonato and R. Nowakowski. The game of cops and robbers on graphs, *Student Mathematical Library, AMS* **61** (2010).

[93] R. L. Breisch. An intuitive approach to speleotopology. *Southwestern Cavers*, 6:72–78, 1967.

[94] R.L. Breisch. *Lost in a Cave-applying graph theory to cave exploration.* National Speological Society, 298p, 2012.

[95] J. Burns, S. Kirtay, and D. Court. Study on administrative and frequency fees related to the licensing of networks involving the use of frequencies. Technical Report 1307/AE/EC/FR/1, European Commission Directorate General Information Society, 2001.

[96] E. Chiniforooshan. A better bound for the cop number of general graphs, *Journal of Graph Theory* **58** (2008), 45–"48.

[97] M. Csüros, A. Milosavljevic. Pooled genomic indexing (PGI) : Mathematical analysis and experiment design. *Lecture Notes in Computer Science*:2452:10–28, 2002.

[98] P. Frankl. Cops and robbers in graphs with large girth and Cayley graphs, *Discrete Applied Mathematics* **17** (1987), 301– 305.

[99] R. G. Downey and M. R. Fellows. *Parameterized Complexity.* Monographs in Computer Science. Springer, 1999.

[100] Martin Farber and Robert E Jamison, *Convexity in graphs and hypergraphs*, SIAM J. Algebraic Discrete Methods **7** (1986), 433–444.

[101] D. J. Guan and X. Zhu. A Coloring Problem for Weighted Graphs. *Inf. Process. Lett.* 61(2):77-81, 1997.

[102] F. Harary and R.A. Melter. On the metric dimension of a graph. *Ars Combinatoria* 2:191-195, 1976.

[103] T. Johnson, N. Robertson, P. D. Seymour and R. Thomas. Directed tree-width. *J. Combinatorial Theory, Ser. B* 82:138–154, 2001.

[104] M. G. Karpovsky, K. Chakrabarty, and L. B. Levitin. On a new class of codes for identifying vertices in graphs. *IEEE Transactions on Information Theory* 44(2):599–611,1998.

[105] W. B. Kinnersley. Cops and Robbers is EXPTIME-complete. *J. Comb. Theory, Ser. B* 111:201–220, 2015.

[106] R. Klasing, N. Morales, and S. Pérennes. On the complexity of bandwidth allocation in radio networks. *Theoretical Computer Science*, **406(3)** (2008), 225–239.

[107] A. Kogan. On the essential test sets of discrete matrice.*Discrete Applied Mathematics* 60(1-3):249–255, 1995.

[108] H. Lehpamer. *Microwave transmission networks: planning, design, and deployment.* McGraw-Hill, 2010.

[109] M. Mamino. On the computational complexity of a game of cops and robbers. *Theor. Comput. Sci.*, 477:48–56, 2013.

[110] T. B. Marcilon, S. Nascimento, and R. M. Sampaio. The maximum time of 2-neighbour bootstrap percolation: Complexity results. In *Proc. of the 40th International Workshop on Graph-Theoretic Concepts in Computer Science (WG)*, volume 8747 of *LNCS*, pages 372–383, 2014.

[111] T. B. Marcilon and R. M. Sampaio. The maximum time of 2-neighbour bootstrap percolation in grid graphs and parametrized results. In *Proc. of the 41st International Workshop on Graph-Theoretic Concepts in Computer Science (WG)*, volume 9224 of *LNCS*, pages 169–185, 2015.

[112] R. W. Payn and D. A. Preece. Identication keys and diagnostic tables : a review. *Journal of the Royal Statistical Society, Series A* 143:253–292, 1980.

[113] A. Quilliot. A short note about pursuit games played on a graph with a given genus, *Journal of Combinatorial Theory Series B* **38** (1985), 89–92.

[114] R. T. Rockafellar, *Convex analysis*, Princeton Mathematical Series, No. 28, Princeton University Press, Princeton, N.J., 1970.

[115] N. Roberson and P. Seymour. Graph minors I–XX. *Journal of Combinatorial Theory Ser. B*, 1983–2004.

[116] A. Scott and B. Sudakov. A bound for the cops and robbers problem. *SIAM J. Discrete Math.*, 25(3):1438–1442, 2011.

[117] B. Shroeder. The cop-number of a graph is bounded by $\lceil \frac{3}{2} genus(G) \rceil + 3$, *Categorical Perspectives, Trends Math.* (2001), 243–“263.

[118] S. Szeider. Not so easy problems for tree decomposable graphs. *CoRR*, abs/1107.1177, 2011.